Alternative non-additively separable utility functions for random utility maximization-based multiple discrete continuous models

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Abstract

Many choice situations involve multiple decisions. For instance, individuals may decide which leisure activities to undertake within a day or which products to purchase while shopping. Moreover, individuals also make a continuous quantity decision for each selected alternative. Bhat (2005, 2008) formulates the multiple discrete continuous extreme value (MDCEV) model that shapes situations where consumers select either one good or multiple goods, along with the decision of how much to consume for each selected good. The implicit assumption of the Bhat's model is that preferences are additively separable, thus preventing the empirical structure from allowing for rich substitution and complementarity patterns in consumption. Bhat, Castro and Pinjari (BCP) (2015) formulate a non-additively separable (N-AS) utility function for the MDCEV model. Such a utility includes interaction parameters that allow the marginal utility of a good k to depend on the quantities consumed of other goods and capture second order interactions. In this research, we start from the BCP's N-AS utility function to explore alternative N-AS functional forms in which the satiation parameters are suppressed in the second sub-utility. The resulting functional forms remains flexible, while also presenting a simple mathematical form of the Jacobian matrix, thus assisting in the estimation procedure.

Keywords

Multiple discrete continuous models, Non-additively separable utility functions, Random utility maximization

Introduction

Many choice situations involve multiple decisions. For instance, individuals may decide which leisure activities to undertake within a day or which products to purchase while shopping. Moreover, individuals also make a continuous quantity decision for each selected alternative. Such multiple discrete continuous (MDC) choice situations have been extensively analyzed in the last decade. Bhat's (2005, 2008) multiple discrete continuous extreme value (MDCEV) model, for example, describes situations where consumers select either one good or multiple goods, along with the decision of how much to consume for each selected good. Additionally, this framework has a nice closed-likelihood expression that simplifies considerably the estimation procedure even in situations with large set of discrete alternatives.

Most MDC studies implicitly assume additively separable preferences, preventing the empirical structure from allowing for rich substitution and complementarity patterns in consumption. Lee and Allenby (2009) formulates an empirical approach that employs a non-additively separable (N-AS) utility function. More specifically, the authors assume that goods within the same group are substitutes while goods in diverse groups are complements. However, the proposed approach does not allow consumers to choose multiple goods within each group. Bhat, Castro and Pinjari (BCP) (2015) formulate a non-additively separable (N-AS) utility function for the MDCEV model. Such a utility can be written as follows:

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \psi_k \gamma_k ln\left[\left(\frac{x_k}{\gamma_k}+1\right)\right] + \frac{1}{2}\sum_{k=1}^{K}\sum_{m\neq k} \theta_{km}\, \gamma_k \gamma_m ln\left[\left(\frac{x_k}{\gamma_k}+1\right)\right] ln\left[\left(\frac{x_m}{\gamma_m}+1\right)\right]$$

$$\text{s.t} \sum_{k=1}^{K} x_k p_k = E \qquad (1)$$

In the Equation 1, x_k refers to the quantity consumed of a good k, ψ_k is the baseline marginal utility at the point at which no goods have been consumed, whereas γ_k allows for corner solutions and accounts for diminishing of marginal utility. The interaction parameters θ_{km} allow the marginal utility of a good k to depend on the quantities consumed of other goods and capture second order interactions. Positive and negative interaction parameters capture complementarity and substitution effects, respectively.

From an economic standpoint, the presence of the satiation parameters in the sub-utility term has no economic implication in capturing complementary and substitution patterns, as such interaction shall primarily depend on the consumption values of goods, and not on their satiation effects, thus making it redundant in the sub-utility term.

In this research, we start from the BCP's N-AS utility function to explore alternative N-AS functional forms in which the satiation parameters are suppressed in the second sub-utility. The resulting functional forms remains flexible, while also presenting a simple mathematical form of the Jacobian matrix, thus assisting in the estimation procedure.

Alternative N-AS functional forms

Two alternative N-AS utility structures are conceived:

A

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \psi_k \gamma_k ln\left[\left(\frac{x_k}{\gamma_k} + 1\right)\right] + \frac{1}{2}\sum_{k=1}^{K}\sum_{m \neq k} \theta_{km} \ln(x_k + 1) \ln(x_m + 1)$$

s.t $\sum_{k=1}^{K} x_k p_k = E$

B

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \psi_k \gamma_k ln\left[\left(\frac{x_k}{\gamma_k} + 1\right)\right] + \frac{1}{2}\sum_{k=1}^{K}\sum_{m \neq k} \theta_{km} \ln(x_k * x_m + 1)$$

s.t $\sum_{k=1}^{K} x_k p_k = E$ (2)

The first alternative functional form looks familiar to the Almost Ideal Demand System (AIDS) model formulated by Deaton and Muellbauer (1980) but, unlike the AIDS's approach, allows for corner solutions and diminishing marginal utility. In the second alternative utility function, complementary and substitution patterns are shaped through a product of pair goods within a single logarithm.

The Karush-Kuhn-Tucker (KKT) conditions for the three functional forms can be written as follows:

Bhat, Castro and Pinjari (2015):

$$\left(\frac{x_k^*}{\gamma_k}+1\right)^{-1}\left[\psi_k+\sum_{m\neq k}^{K}\theta_{mk}\gamma_m \ln\left(\frac{x_m}{\gamma_k}+1\right)\right]-\lambda p_k=0 \; if \; x_k^*>0, \text{ k=1,…,K}$$

$$\left(\frac{x_k^*}{\gamma_k}+1\right)^{-1}\left[\psi_k+\sum_{m\neq k}^{K}\theta_{mk}\gamma_m \ln\left(\frac{x_m}{\gamma_k}+1\right)\right]-\lambda p_k<0 \; if \; x_k^*=0, \text{ k=1,…,K}$$

First alternative formulation:

$$\left(\frac{x_k^*}{\gamma_k}+1\right)^{-1}\psi_k+\sum_{m\neq k}^{K}\frac{\ln(x_m+1)\theta_{mk}}{x_k+1}-\lambda p_k=0 \; if \; x_k^*>0, \text{ k=1,…,K}$$

$$\left(\frac{x_k^*}{\gamma_k}+1\right)^{-1}\psi_k+\sum_{m\neq k}^{K}\frac{\ln(x_m+1)\theta_{mk}}{x_k+1}-\lambda p_k<0 \; if \; x_k^*=0, \text{ k=1,…,K}$$

Second alternative formulation:

$$\left(\frac{x_k^*}{\gamma_k}+1\right)^{-1}\psi_k+\sum_{m\neq k}^{K}\frac{x_m\theta_{mk}}{x_k+1}-\lambda p_k=0 \; if \; x_k^*>0, \text{ k=1,…,K}$$

$$\left(\frac{x_k^*}{\gamma_k}+1\right)^{-1}\psi_k+\sum_{m\neq k}^{K}\frac{x_m\theta_{mk}}{x_k+1}-\lambda p_k<0 \; if \; x_k^*=0, \text{ k=1,…,K} \qquad (3)$$

It is important to note that large values for negative interaction coefficients may lead the marginal utility to become negative, which is not consistent with economic theory and can cause estimation difficulties. To address this issue, Bhat, Castro and Pinjari (2015) impose conditions during model estimation that the marginal utility at observed consumption values be positive for all choice alternatives and all individuals in the data. To the extent that the parameter space is restricted by such conditions, it may be difficult to estimate rich substitution patterns from empirical datasets. In the proposed research, we count the number of individuals (and alternatives) in the empirical dataset with have negative marginal utility at the estimated parameters. This number can be employed as a method to compare the alternative functional forms to the BCP’s utility structure.

Data

The proposed utility functions and the one developed by Bhat, Castro and Pinjari (2015) are applied to the 2002 Consumer Expenditure (CEX) Survey conducted by the U.S. Census Bureau for the Bureau of Labor Statistics (also used in BCP, 2015). The data comprises information about individual and household socio-economic and employment as well as expenditures in 109 categories. In the analysis, we focus on expenditures in six transportation categories: (a) Vehicle purchase, (b) Gasoline and motor oil, (c) Vehicle insurance, (d) Vehicle operation and maintenance, (e) Air travel, and (f) Public transportation. The remaining household expenditures are gathered in the *essential outside good* category.

Empirical findings

Overall, the estimation findings are intuitive (Table 1). Also, while the empirical models from all three utility functions provide largely similar interpretations, there are differences in the estimated interaction parameters. More specifically, larger household expends a lower proportion of income on Air travel and Public transportation, presumably because of money budget constraint. As the number of vehicles in the households increases so does the proportion of income allocated to all the expenditure categories, except on Public transportation. Those families who live in an urban area expend a greater proportion of their income on Air travel compared to those who live in a non-urban area, probably because of the better air connectivity of the urban areas. Focusing on the interaction effects, the second proposed utility function results in a significant substitution effect (i.e., negative interaction parameter) between Vehicle purchase and Public transportation, perhaps because those people who spend a larger portion of their budget on Vehicle purchase are less inclined to use public transport modes. Interestingly, the empirical model with the BCP's functional form estimates a significant complementary effect (positive interaction parameter) between Vehicle purchase and Public transportation expenditures, which appears to be counterintuitive (similar result is observed in Bhat, Castro and Pinjari, 2015).

Table 1 : MDCP RESULTS

Variables	BCP		ALT1		ALT 2	
	Parameter	t-stat	Parameter	t-stat	Parameter	t-stat
Baseline Utility Parameters						
Baseline Constants						
Vehicle purchase	-5.03	-97.47	-4.73	-82.74	-4.53	-87.62
Gasoline and motor oil	-2.19	-38.54	-2.05	-32.77	-2.19	-38.80
Vehicle insurance	-3.01	-73.15	-2.76	-56.90	-3.01	-71.62
Vehicle operation and maintenance	-2.86	-71.16	-2.39	-43.92	-2.65	-60.37
Air travel	-4.45	-97.43	-3.73	-63.27	-4.44	-96.93
Public transportation	-4.32	-51.55	-4.25	-50.14	-4.25	-52.78
Number of workers in household						
Vehicle purchase	0.07	2.75	0.05	1.78	0.09	3.80
Gasoline and motor oil	0.18	11.27	0.14	8.10	0.18	11.36
Vehicle insurance	0.08	4.12	0.10	5.50	0.08	4.09
Vehicle operation and maintenance	0.15	8.23	0.16	8.94	0.18	9.42
Annual household income 30-70K (<30 is base)						
Vehicle purchase	0.63	10.63	0.63	10.61	0.70	12.08
Gasoline and motor oil	-0.07	-2.34	0.00	-0.06	-0.07	-2.45
Air travel	0.57	10.40	0.62	11.27	0.57	10.39
Annual household income >70K (<30 is base)						
Vehicle purchase	0.54	7.19	0.47	6.07	0.51	7.30
Gasoline and motor oil	-0.65	-15.89	-0.53	-12.59	-0.65	-16.67
Vehicle purchase	-0.41	-11.10	-0.38	-10.79	-0.41	-10.95
Air travel	0.71	11.05	0.67	10.32	0.71	11.07
Number of vehicles in household						
Vehicle purchase	0.13	9.39	0.14	10.83	0.14	10.76
Gasoline and motor oil	0.26	21.42	0.25	20.39	0.26	21.42
Vehicle insurance	0.23	17.31	0.23	17.37	0.23	17.29
Vehicle operation and maintenance	0.19	14.03	0.22	15.93	0.23	16.17
Air travel	0.02	1.15	0.10	7.14	0.02	1.16
Public transportation	-0.08	-4.47	-0.08	-4.14	-0.03	-1.71
Satiation Parameters						
Vehicle purchase	58.9	308.4	36.00	200.1	32.50	219.7
Gasoline and motor oil	0.40	24.7	0.40	23.6	0.40	24.8
Vehicle insurance	0.90	36.0	0.80	34.3	0.90	35.7
Vehicle operation and maintenance	0.70	29.8	0.50	26.6	0.60	35.1
Air travel	1.30	24.8	1.00	26.4	1.30	24.8
Public transportation	0.60	20.1	0.60	19.5	0.60	25.9
Interaction parameters						
Vehicle purchase and Vehicle operation and ma.	$7*10^{-4}$	10.71	$34*10^{-3}$	7.3178	$-29*10^{-3}$	-30.96
Vehicle purchase and Air travel	$15*10^{-3}$	22.02	$-64*10^{-3}$	-7.962	$-21*10^{-3}$	-22.02
Number of parameters	35		35		35	
Log-Likelihood at convergence	-37955.29		-38017.95		-37795.42	

In terms of goodness of fit, it appears from Table 2 that the Second alternative utility function has a better model fit as evidenced by lower BIC values with respect to the other two functional forms.

Table 2: MODEL FIT RESULTS

Empirical frameworks	Log-likelihood at convergence	# Parameters	# Observations	BIC
Bhat, Castro and Pinjari (2015)	-37955.29	35	3599	38098.58
First alternative utility function	-38017.95	35	3599	38161.25
Second alternative utility function	-37795.46	35	3599	37938.76

Conclusions

To summarize, the proposed research aims at exploring alternative N-AS utility functions that shape consumption situations where goods can be either complements or substitutes to each other. The preliminary investigation reveals that the first alternative formulation offers a superior performance over the BCP utility function form, suggesting that the suppression of satiation parameters in the second sub-utility takes better account of MDC consumption decisions. The next step will be to compare the proposed functional forms based on: (1) the space of parameters and consumptions where marginal utility is not negative and (2) comparisons of interpretations, model fit, and runtime on additional empirical data sets.

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