Modeling Uncertainty for a Catenary-free Electrical Bus

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Conference paper STRC 2016

STRC | **16th Swiss Transport Research Conference**
Monte Verità / Ascona, May 18-20, 2016

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April 2016

Abstract

Air pollution is a major problem of the mobility in large cities since particulate matter and nitrogen oxide can cause severe health problems. Any means of transportation dependent on fossil fuels contributes to these problems. Regarding public transportation, buses cause a huge amount of the emissions. Electric buses provide a locally emission-free alternative that can also reduce the overall emission of greenhouse gases. Recent innovations in battery technology allowed the construction of battery-driven electric buses that require charges at distinguished stops, rather than catenaries. The demonstration project TOSA in Geneva showed the feasibility of this approach.

A major issue of a catenary-free bus system are the relatively high expenses. Therefore, cost optimization of these systems is a major key for their promotion. The existing literature is scarce and largely focuses on the optimization if the parameters of the system (e.g. the energy consumption) are certainly known in advance. These models, however, are not sufficient as results may not be feasible due to perturbations in the system or extremely costly as a result of worst-case assumptions. Hence, in this paper both a straight-forward “conventional” model and for the case with uncertainty, a new greedy heuristic approach to the problem is developed. The former optimizes the installation cost of the charging stations and the battery size. The latter incorporates the uncertainty in energy consumption of the bus, but suffers from the limitation that only one station type can be considered and that the determination of the battery size is sub-optimal.

A case study is carried out to compare the conventional and the robust model using partly real-world data. The results reveal a remarkable cost increase by adding the uncertainty. The increase strongly depends on the assumptions about the uncertainty. The greedy property of the algorithm, however, does not decrease much the solution quality.

Keywords

Tactical design, Facility location, Electrical Bus, Feeding station, Mathematical model, Heuristic

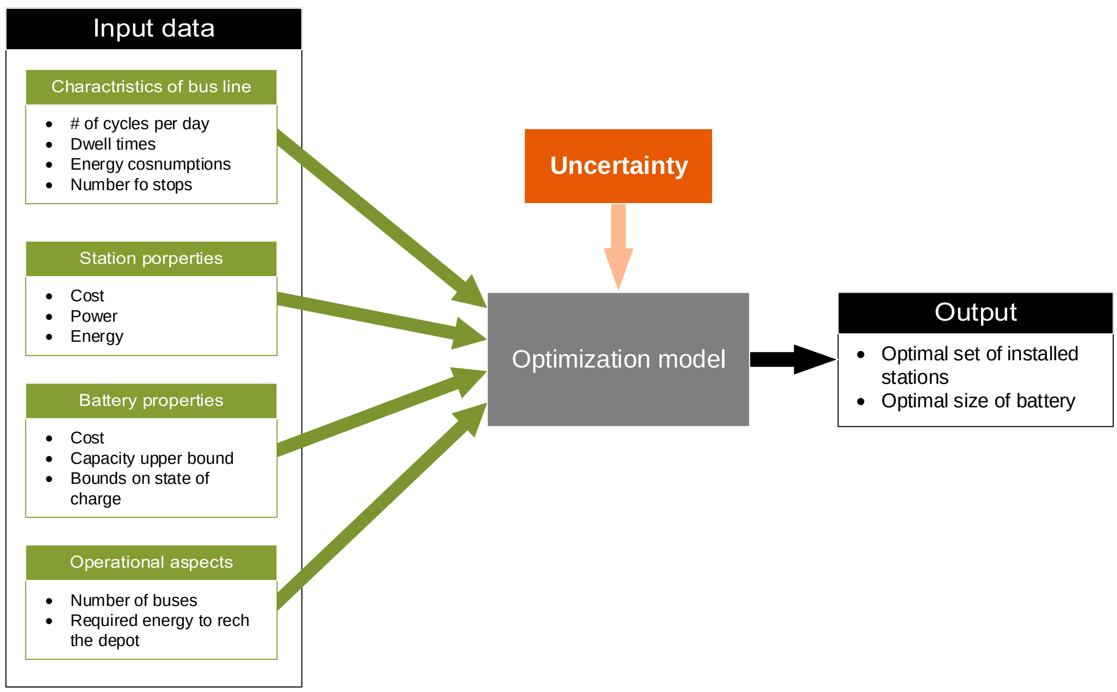
1 Introduction

Air pollution is a major problem in large cities all around the world. Even if the major part of this pollution is produced by private transport, public transport also contributes to these problems. Electrical buses can be an environmental-friendly alternative to diesel-driven ones. In some cities, trolleybuses already operate for decades. However, cities wishing to introduce an electric bus system do not want to install overhead lines all over the city. Catenary-free electric buses can overcome this problem. Since the main cost drivers of electrical buses are the battery in the buses and the charging infrastructure, this paper addresses the optimization of the battery size and the placement of charging stations. The stations can have different types and are placed at locations where the bus stops anyway, following the opportunity charging concept.

First, a classical optimization approach is presented based on the literature. Because some parameters of the optimization model depend on the traffic state and the passenger volume, conventional modelling does not seem adequate for the problem at hand. Thus, this paper presents a heuristic approach capturing the uncertainty. The most important source of uncertainty is the energy consumption that can be affected by passenger volume, the traffic state and several other factors being highly uncertain.

The inputs and outputs of the model are sketched in Figure 1. First, a model is developed for the deterministic approach.

Figure 1 Input and output of the optimization model to be developed in this paper



2 Literature review

In this paper, we solve a facility location problem for electrical buses. We formalize the mathematical model and develop a heuristic in order to capture uncertainty.

The objective of this paper is to develop a model to **optimize the location** of charging stations and the battery capacity for an **electrical bus line** for both the case without and with uncertainty. The next paragraphs review the relevant literature to these aspects.

Optimization of bus networks mostly focuses on network design problems where the objective usually is the maximization of social benefit with respect to budget or rolling stock constraints (Desaulniers and Hickman 2007). For the problem at hand, another approach has to be taken since the problem is more similar to facility location problems. The facility location problem is a well-studied mixed integer linear model. Owen and Daskin (1998), Daskin (2008) and Farahani et al. (2012) are some of the few references for facility location problems. However, the problem at hand has a different structure. The most important difference is the necessary consideration of the energy in the battery that is recharged at the stations and discharged on the route. Consequentially, the energy consumption of the bus has a main influence on the design of the network.

Optimization models for refueling (charging) of private vehicles are addressed by Wang and Lin (2009) and Wang (2011) whose models explicitly calculate the remaining range (equivalent to remaining energy) at each node of a street network. The authors refer to the general case of many different paths, but with homogenous vehicles. The remaining range when arriving at each node of the network is calculated out of the energy when arriving at the previous node plus the refueled energy at that node minus the energy consumption between the nodes. The charged energy is limited by charging time and power, and by the fuel tank (e.g. battery) capacity. In Wang and Lin (2013), the model is extended to more than one station type. The types differ by cost, by charging power and by the number of vehicles that one station can serve. However, there are many stations (of different or the same type) possible at the same node. However, for the problem at hand, at each stop, only one charging station can be installed anyway. The paper by Wang and Lin (2013) can be modified for the application to electrical bus lines

However, there are some publications that optimize bus charging infrastructures by considering their particularities. Kunith et al. (2014) present a model that minimizes the installation cost for the fast-charging stations of an urban electrical multi-line bus system. Their model tracks the energy in the battery and assures that the battery energy always stays into the specified bounds. The charging process is modeled with a nonlinear function in order to account for the fact that the power during charging is usually not constant. However, the approach is relatively simple using generally the same approach as Chen et al. (2013) whose approach will be presented in the next paragraph. On the contrary, their mathematical model is becomes relatively complex because of the piecewise linear charging function and the usage of Big M method in many constraints. That is why in this paper, constant power is assumed during the charging process.

Chen et al. (2013) present a mixed-integer program with a similar idea, but with a linear charging function and with two different types of charging stations. They also consider a single electrical bus line, including the optimization of the charging stations and the size of the battery. The authors assume all cycles to be identical which will be similar in the model in this paper. Additionally, they impose the constraint that the battery has to be fully charged at each of the two terminals before beginning a trip. This assumption will not be made in this paper. The stations in their model are differentiated between slow charging stations that only have a converter, and fast ("flash") charging stations that also have an energy storage to buffer energy for the recharge of the buses. For the latter type, the model assures that the energy storage will be fully recharged between two subsequent buses by assuming a constant headway between all buses. Additionally, charging at the fast charging stations with energy storage is assumed to be done instantaneously. Because Chen et al. (2013) consider the two types of charging stations, there are two Boolean decision variables for each stop: One determines if there is a station and the other one indicates whether there is a station with energy storage. Some other continuous decision variables describe properties of the charging stations that are determined independently for each station. In this paper, the assumptions are different: A finite set of charging stations is considered so that there needs to be a decision variable for each stop and each station type. This makes the choice of the stations more realistic since the station manufacturer will not produce stations where each station has a different converter or energy storage.

The authors also include the way from the depot into their model and they add a constraint that assures that the bus can reach the depot from every stop, e.g. if there is a major issue with the on-board charger of the battery. The depot will not be considered in our model as in the first cycle of the day, the energy consumed on the way between depot and terminal can be recharged. In order to always be able to reach the depot, a fixed value for the minimum energy in the battery is added.

Besides the determination of the station locations and the respective properties, the bus battery size and the size (power) of the on-board charger of this battery is also determined by their optimization model. The former is included into the model in this paper. On the contrary, the latter will not be added to the model because its size is assumed to be given by the charging station type with the highest power.

To the best knowledge of the authors of this paper, there is no model in the literature yet dealing with the optimization under uncertainty of bus charging infrastructures. This paper contributes to closing this gap.

3 Modeling

3.1 Assumptions

The assumptions of the model are listed in the following and explained in the subsequent paragraphs.

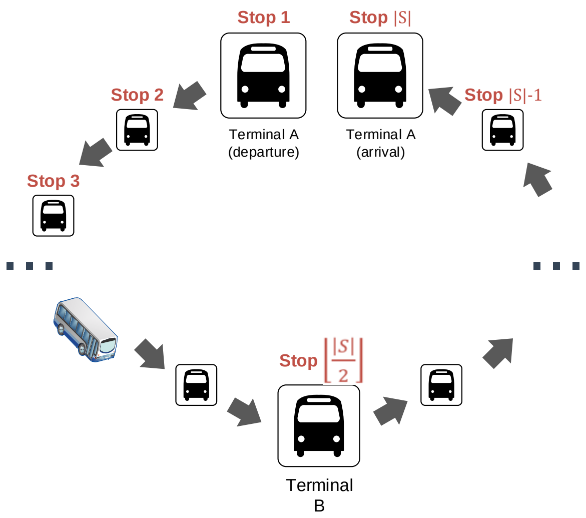
- Type of bus line
- Charging stations
- Daily cycle and battery

Bus line

Similar to Chen et al. (2013), the considered bus line is assumed to be a circle route, starting and ending at the terminal stop. The consideration of different origin-destination pairs for both ways of a normal linear route as done by Wang and Lin (2013) has not been chosen for modeling convenience.

The round line can also be interpreted as a normal one by considering each stop twice, namely for back and forth direction. However, there is no difference in the models if the following assumption is made: There is no gain in installing charging stations for both directions at the same stop instead of installing them at different stops.[1] Nevertheless, the assumption for the case study conducted in a later part of this paper (see section 4) is that no such synergies exist and that there are two terminals as sketched in the following figure where $|S|$ denotes the number of stops in the model.

Figure 2 Bus line considered in this paper



[1] The mixed integer model, being presented in the next chapter, could also be modified easily to account for these synergies.

Terminal A is considered twice: both as the first and the last stop. In the middle of the bus line, there is the second terminal that is treated like a normal stop with a longer dwell time. Thus, physically there are $|S| - 1$ stops of which two are terminal stops.

Charging stations

Charging stations at the stops can have different types. For the energy charged at a station, it is important to state that there may be two limitations: First, charging power is limited due to limitations of the converter or the connection to the electrical grid. Secondly, the charged energy can be limited by the capacity of an energy storage that is part of the charging station. The latter limitation is not made by Wang and Lin (2013) because their model mainly focuses on charging stations for individual mobility that usually are not equipped with a buffer storage. The former limitation is due to the fact that the charging time is limited by the schedule. The product of power and time is energy, so the charged energy is limited by the charging power and the dwell time at the stop. To overcome limitations due to the electrical grid, an energy storage can be installed in the charging station. This internal storage is then charged over a relatively long time (usually the headway between two subsequent buses) and discharged in a few seconds when the bus is at the stop (Chen et al. 2013). This imposes the latter constraint: The energy that can be charged to the bus is limited by the capacity of the energy storage in the station. Additionally, the latter constraint can be used to implement an upper bound on the charging time (e.g. to avoid overheating of the charger).

The model presented in Chen et al. (2013) considers for each station the energy storage size and the power of the charger as decision variable. Thus, there is theoretically an infinite number of charging station types, which does not seem reasonable because it is relatively costly to produce many different station types. That is why it is assumed that there is a limited number of station types, each with different cost, power limit and energy limit (due to the internal storage). Additionally, the authors did not impose a power limit for the stations with energy storage because they assumed that the charging process is instantaneous (“flash” charging). Because this is not physically possible (and thus there is also a power limit for this kind of charging station) a power limit for all stations is part of the model. Consequently, the energy that can be charged by a station with energy storage is limited in both energy and power. On the contrary, the energy charged by a station without this kind of storage is only limited in power. For modeling convenience, both constraints will be implemented for all station types. The energy limit of stations without energy storage is thus set to the upper limit of the battery capacity.

Daily cycle and battery usage

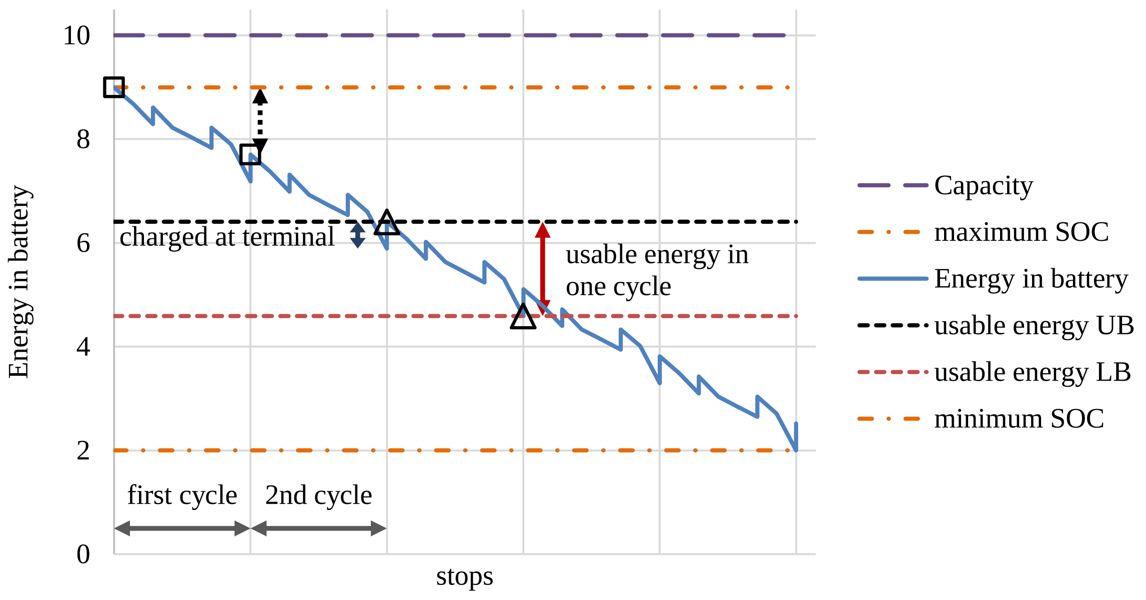
According to Chen et al. (2013), the vehicles are assumed to be charged slowly over night so that the assumption of a fully charged battery at the beginning of a day seems to be reasonable. The cost of this charger is not included in the model because it will be built anyway and the type of this station does not depend on the installations at the stops. Additionally, the cost of this charger will probably differ from the cost of the chargers at the stops since it is built in the depot and has several charging points.

The battery capacity is assumed to be the same for all buses because of homogeneity of the bus fleet for the considered line. Additionally, the battery capacity can be chosen from a continuous interval.

Every cycle is assumed to be identical so that it is sufficient to only consider one cycle. Nevertheless, not the whole battery range, from 0 % to 100 % state of charge (SOC), can be used during this cycle:

(1) For fast charging, not the whole battery range is available. Depending on the cell technology, the last 10 % of the battery capacity cannot be used for fast charging. (Rogge et al. 2015)
(2) The battery capacity drops during lifetime. A battery is claimed to be defect if the remaining battery capacity drops below 80 % of the initial value. (Rogge et al. 2015)
(3) Due to overnight charging, the battery can be assumed to be full at the beginning and empty at the end of the day[2]. At the terminal stations, the battery might only be partly recharged. Thus, the battery is sequentially discharged throughout the day as outlined in Figure 3.

Figure 3 Evolution of energy in battery during the day (illustrative example)



[2] Chen et al. (2013) force the battery to be fully charged when each cycle starts. This seems to be a too hard assumption since it might be beneficial to choose a battery capacity bigger than the capacity that is needed for one cycle in order to save charging stations.

The dotted orange lines represent upper and lower bound for the energy that can be used during the whole day due to (1) and (2). To calculate the energy that is usable during one cycle, it is assumed that a station is built at least at terminal stop A. This seems to be reasonable because there are usually higher stopping times at terminal stops. The energy charged at this station has to be fixed in advance which is not considered as a problem since there will be a station without energy storage installed anyways[3].

The energy that is consumed in the first cycle is the difference between the small squares in Figure 3. Intuitively, one could say that the usable energy is the energy between the orange lines divided by the number of cycles per day. That is not right because then, during the last cycle, energy would drop below the minimum level (2 kWh in Figure 3). That is why the amount of net energy 'lost' in each cycle (black dotted arrow) is not the difference between minimum and maximum SOC divided by the number of cycles. On the contrary, the usable energy in one cycle (red arrow) is the dotted arrow plus the energy recovered at the terminal charger. It is denoted by c:

$$c = \frac{w(\kappa - \zeta) - \pi}{\lambda} + \pi \tag{1}$$

The first term in the nominator accounts for the upper and lower bound during the day: w is the physical capacity of the battery (decision variable), κ and ζ are upper and lower bound for the energy usable throughout the day, measured in % of the battery capacity. π is the energy charged at the terminal stop and λ is the number of cycles. This energy is represented by the red arrow in Figure 3 which is the difference between the energy when starting the cycle (the first triangle) and the energy at the end of the cycle before having charged at the terminal (second triangle). In the time at the terminal, π is charged.

For readability reasons, the minimum energy is not shown in the figure. This energy is necessary to assure to be able to reach the depot even if there is a major problem, e.g. a defect of the on-board battery charger. This could be considered by changing equation (1) to:

$$c = \min\left\{\frac{w(\kappa - \zeta) - \pi - \nu}{\lambda} + \pi\, , w(\kappa - \zeta) - \nu\right\} \tag{2}$$

where ν is the energy required to reach the depot from the stop that is the farthest away from the depot. What happens is that not the usable energy per cycle is reduced by ν, but the usable energy during the day. The second term in the minimum expression accounts for the possibility that a relatively high amount is charged at the terminal ($\pi > w(\kappa - \zeta) - \nu$).

[3] If there was a station with energy storage at the terminal, this storage would be costly due to the relatively high amount of energy charged, and there would be not much time to recharge this storage because this station would be occupied for a relatively long time.

3.2 Conventional model

Notations

The notation listed in the following will be used in both the deterministic and the robust model.

Sets

Stops: $s \in S = \{1, \dots, |S|\}$

Charging station types: $t \in T = \{1, \dots, |T|\}$

Parameters

α_s^t Cost of installation of a station of type t at stop s (CHF)

β Specific cost of bus battery (CHF/kWh)

γ Number of buses

ζ Lower bound for energy in battery (in % of battery capacity)

κ Upper bound for energy in battery (in % of battery capacity)

λ Number of cycles per day

μ_s Energy consumption between stop s and $s+1$ (kWh)

ν Energy required to reach the depot from the stop that is the farthest away from the depot

θ_s Dwelling time at stop s (hours)

π Energy charged at terminal charger (kWh)

ρ_s^t Power of charging station type t if installed at stop s (kW)

ϕ_s^t Energy limit for one charging process at station type t if installed at stop s (kWh)

$\overline{w}$ Upper bound for battery capacity (kWh)

Decision variables

w Battery capacity (kWh)

c Usable battery capacity (kWh)

$$x_s^t = \begin{cases} 1, & \text{if station of type } t \text{ is installed at } s \\ 0, & \text{otherwise} \end{cases}$$

y_s Charged energy at stop s (kWh)

z_s Energy in battery when reaching stop s (kWh)

Note that the parameters cost (α_s^t), power (ρ_s^t) and energy limit (ϕ_s^t) for each station type can differ between the stops. In this way, it can be accounted for differences in the construction and the grid connection at different sites.

It can be seen that the number of decision variables grows linearly with the number of stops and the number of station types.

Mathematical model

In this subsection, the mixed integer model for the deterministic case is presented. The basic idea taken from Wang and Lin (2013) and Chen et al. (2013) is to calculate the charged energy at each stop and the energy when reaching the stop.

In the following, the mixed integer model is presented and the constraints are explained afterwards.

$$\min_{w,c,x_s^t,y_s,z_s} \; w\,\beta\,\gamma + \sum_{s\in S}\sum_{t\in T} \alpha_s^t\, x_s^t \tag{3}$$

$$\text{s.t.} \quad z_{s+1} \le z_s + y_s - \mu_s \qquad \forall s \in S \tag{4}$$

$$y_s \le c - z_s \qquad \forall s \in S \tag{5}$$

$$y_s \le \sum_{t\in T} x_s^t\, \theta_s\, \rho_s^t \qquad \forall s \in S \tag{6}$$

$$y_s \le \sum_{t\in T} x_s^t\, \phi_s^t \qquad \forall s \in S \tag{7}$$

$$\sum_{t\in T} x_s^t \le 1 \qquad \forall s \in S \tag{8}$$

$$c \le \frac{w(\kappa - \zeta) - \pi - \nu}{\lambda} \tag{9}$$

$$c \le w(\kappa - \zeta) - \nu \tag{10}$$

$$0 \le w \le \overline{w} \tag{11}$$

$$x_s^t \in \{0,1\} \qquad \forall s \in S, t \in T \tag{12}$$

$$y_s \ge 0 \qquad \forall s \in S \tag{13}$$

$$z_s \ge 0 \qquad \forall s \in S \tag{14}$$

The objective function (3) minimizes total cost that is composed of the battery cost for all vehicles and the installation cost. In line with Chen et al. (2013) and Wang and Lin (2013), the next two constraints are formulated: constraints (4) are to calculate the energy in the battery when arriving at each station out of the energy at the previous station and the charged and consumed energy in between the stations. Constraint set (5) limits the charged energy to the difference between the maximum energy (the usable energy c) and the energy before charging. The next two constraints are different to the model from Chen et al. (2013) as it has already been explained above. The constraint sets represent limitations of the charged energy because of the station properties: (6) is the limitation because of power and dwell time. The other one (7) is necessary for the stations that have an energy storage: There, the energy is limited by the energy storage capacity. If there is no energy storage in the station, ϕ_s^t is set to a sufficiently large value, namely the upper bound for the battery energy $\overline{w}$. The constraints (6) and (7) also assure that there is no energy charged at a stop if no station is built there. Inequalities (8) guarantee that there is at most one station at each stop. Inequalities (9) and (10) are the linear formulation of (2) linking the real battery energy w and the usable energy c. The bounds of the battery capacity are written in inequality (11). Equation (12) is the set of integrality constraints. Non-negativity of the other continuous variables is assured by (13) and (14).

It can be seen easily that the number of constraints grows linearly with the number of stops.

3.3 Robust model

Some of the parameters of the above presented model are subject to uncertainty. The most important ones are the energy consumptions of the segments of the bus route which are considered in the following.

Assumptions about the uncertainty

The assumptions about the uncertainty are very similar to these stated by Bertsimas and SimBertsimas and Sim (2004)(2004). Since the distribution of the parameters is unknown, only a interval of the energy consumptions is defined where μ_s is the nominal energy consumption and ϵ_s is the maximum deviation:

$$[\mu_s, \mu_s + \epsilon_s] \ \ \forall s \in S \tag{15}$$

In order to facilitate the interpretation of the uncertainty, a parameter χ ($0 \leq \chi \leq 1$) is introduced that indicates the relation between the deviation and the mean energy consumption in one segment:

$$\epsilon_s = \chi \, \mu_s \ \ \forall s \in S \tag{16}$$

The uncertainty set for the energy consumption in segment s can now be formulated as:

$$[\mu_s - \epsilon_s, \mu_s + \epsilon_s] = [\mu_s(1-\chi) \, , \mu_s(1+\chi)] \ \ \forall s \in S \tag{17}$$

An approach similar to the budget of uncertainty approach of Bertsimas and Sim (2004) is used: The idea is that it is very unlikely that there is high passenger load, extremely low speed and deviations in all parts of the bus route. In our case, the above defined interval (17) incorporates the effect of these events. The budget of uncertainty is the number of segments where the energy consumptions is higher than the nominal value. Until stop $s+1$, there are s energy consumptions. That is to say that with s the number of segments grows, in which the energy consumption can be either high or low. Consequently, the budget of uncertainty (denoted by Γ) needs to be different for each s. By introducing a parameter ψ that is the fraction of stops where the energy consumption can differ in an unfavorable way, we can write the budget of uncertainty:

$$\Gamma_s = \min\{ \lfloor \psi(|S|-1) \rfloor \, , s \} \quad \forall s \in S \backslash \{|S|\} \tag{18}$$

Until stop $\lfloor \psi(|S|-1) \rfloor$, all energy consumptions are assumed to have the worst-case value. Afterwards, this is the case for $\lfloor \psi(|S|-1) \rfloor$ of the energy. The floor function is necessary since there can only be integer number of segments with high energy consumption in the model.

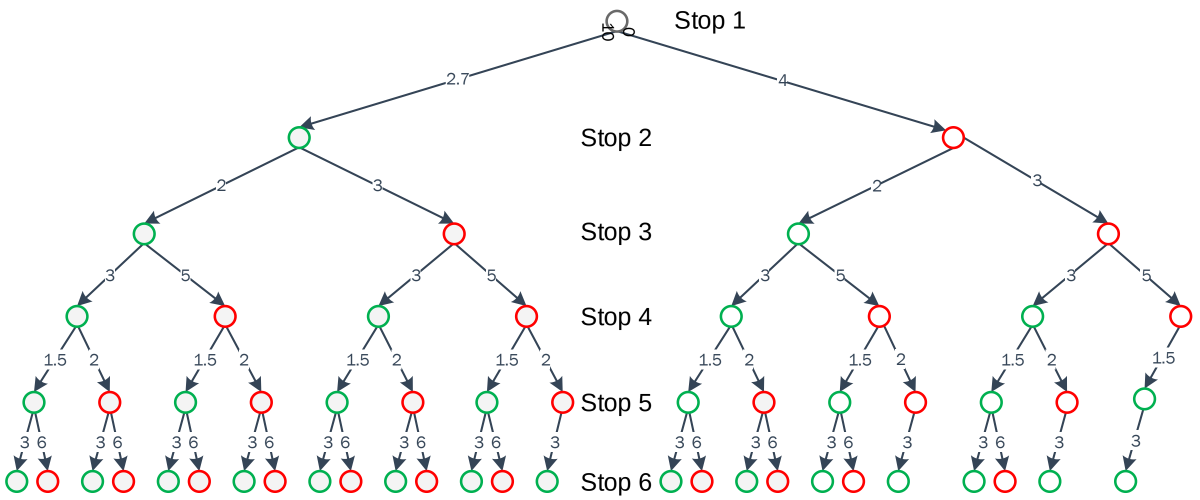
The interpretation of ψ and χ is as follows: ψ is the fraction of segments of the bus route where the energy consumption can be higher than the nominal value at the same time. The amount by which the energy consumption is raised is the fraction χ.

Model

Because it is not clear in advance, which combinations of high and normal energy consumptions are the critical ones, all combinations are enumerated. A modification of the deterministic model is not possible because of the large number of possible combinations that would have to be included into the model. This would lead to a massive increase of constraints and decision variables. (The number of combinations raises exponentially with the number of stops.) Thus, a simple heuristic approach is used that is described in the following.

The concept is to enumerate all combinations of normal and high energy consumption that comply to the budget of uncertainty. This is to say that the number of sections with elevated energy consumption is limited by the budget of uncertainty. The enumeration is implemented in the form of a tree where each branch corresponds to high or normal energy consumption as it is illustrated in Figure 3 with fictive data and only a few stops.

Figure 4 Visualization of the tree used for the heuristic. Red cycles represent high energy consumption and green ones nominal energy consumption. (Illustrative example)



The tree is pruned on the right because in this example, the budget of uncertainty which has been chosen as $\Gamma_s = \min\{3, s\}\ \forall s \in S \backslash \{|S|\}$, is exceeded there. The numbers on the arrows are the energy consumptions corresponding to high and normal energy consumption respectively.

The heuristic is not able to directly determine the optimal capacity which is one of our objectives. Consequently, the heuristic is run several times for different battery capacities. The capacity with the lowest overall cost is then chosen.

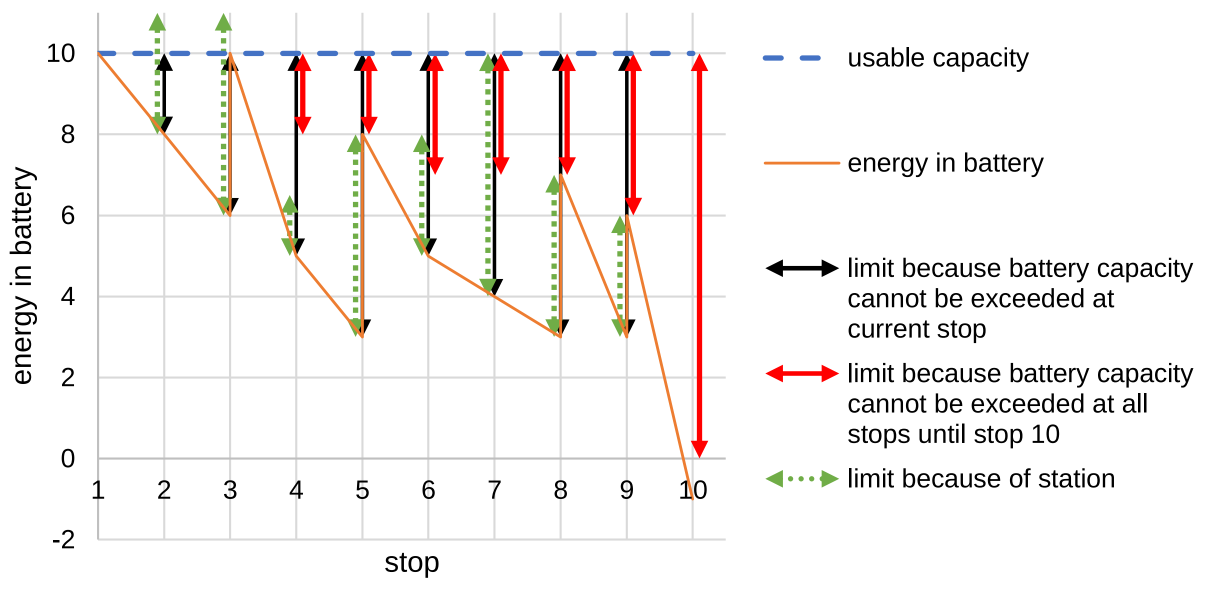
After the construction of the tree, the stations need to be placed. For this, each node is visited by depth-first search, beginning with the root. The battery energy when leaving the first stop is the usable energy being calculated according to equation (2).

Because the battery will run out of energy faster if the energy consumption is higher, the branches with high energy consumption are visited first. When visiting a station, the energy in the battery is calculated out of the energy when leaving the predecessor and the energy consumption to the node.

When the energy drops below zero, an additional station needs to be placed at an earlier stop. A greedy algorithm places the stations after having determined the chargeable energy at each stop if there was an additional station there. (The exact procedure is explained in the next paragraphs.) A station is then built at the stop with the highest chargeable energy. Because the greedy algorithm can only compare chargeable energies and the cost related to an additional installation is not considered, only stations of one type that have the same cost at every stop can be built.

The energy that could potentially be charged is calculated by going backwards through the graph. The procedure is illustrated in Figure 5 for fictive data.

Figure 5 Illustrative example for the determination of the chargeable energy



The figure shows an example where in a certain path in the tree, the energy at the arrival at stop 10 drops below zero. The algorithm then needs to put a charging station at a stop prior to stop 10. It backtracks towards the root of the tree through all stops $s \in \{1,..,9\}$. Each time it calculates the energy that will be additionally in the battery at stop 10 if there was an additional station at s. First, this energy is limited by the limit of the potentially installed station at s (e.g. because of charging power). The dotted green arrows in Figure 5 represent this limit. Secondly, the energy at each stop after the installation cannot exceed the usable capacity. The maximum charged energy has to be smaller than the gap between the usable capacity (blue dotted line) and the energy of the battery at each stop $s' \in \{s, \dots, 10\}$. This constraint is shown as red arrows in the figure. It can easily be seen that this gap becomes smaller during backtracking. Third, if there is already a station, the additional chargeable energy is zero for this stop.

If the gap between usable capacity and the energy in the battery becomes zero because the battery is already full when leaving a certain stop, the backtracking is stopped and the different additional energies are evaluated. At the one with the highest value, the new station is installed. The energies at the stops must then be calculated again, beginning from the node where the station has been placed. Note that after the installation of a new charging station, not all energies are calculated again. That is not necessary because for the paths that have been already calculated totally, the installed stations without this new station were already sufficient.

When all nodes have been visited, the algorithm has found a feasible solution. The same procedure is repeated for different battery capacities in order to determine the optimal capacity.

4 Case study

In this chapter, we apply synthetic data inspired from a real case study to validate and test the models. The aim of this case study is not to provide specific results, but to assess the quality of the models presented in the last chapter.

Both models are implemented in C++ by the help of Microsoft Visual Studio (32 bit). For the optimization, CPLEX 12.6.2 (32 bit) is used. The calculation is carried out on an Acer notebook with an Intel Core i5-2430M processor and 4 GB RAM. CPLEX can use two threads and the CPU speed is limited to 1.68 GHz.

4.1 Input data

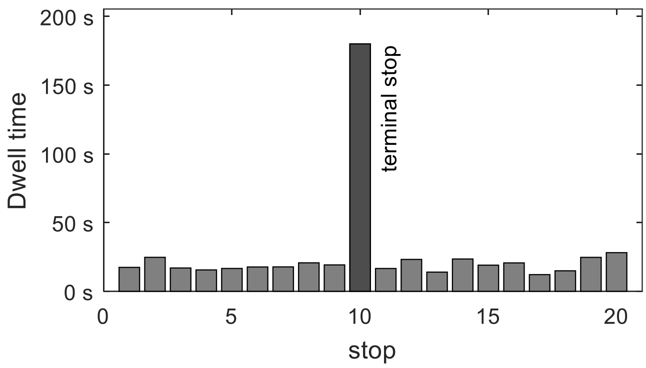
In the following paragraph, the data used for the calculation of all models is presented.

The **energy consumptions** are calculated based on the mean of some data from a major supplier of bus charging stations. This value is then superposed with a bell-shaped distribution in order to account for different distances between the stops and passenger load at each segment of the line.

In the literature, there exist different values for mean **dwell times**. Tirachini (2011) reviews studies on dwell times which model the dwell time as a function of passenger load. The part of the dwell time that is independent from passenger load is varying between 5 and 16 s for recent studies. For each passenger, the studies report 0.4 to 6 seconds as additional time.

For this case study, however, there is no passenger load data available. Thus, the dwell times are modeled as realizations of a normally distributed random variable. The realizations are calculated in advance and the same values are taken for all calculations. For the mean value, 20 s is used and 4 s is taken as standard deviation. Because the considered line is assumed to have two terminal stops of which one is in the middle of the route, this stop can be assumed to have a longer dwell time. The dell time at this stop is assumed to be 180 s. The dwell time for all segments is shown in the following figure.

Figure 6 Dwell time used in the case study



Each **station type** has three properties: Cost, and the limit of energy and power. There are three different station types:

(1) Slow charging station (standard)
(2) Slow charging station (higher power)
(3) Fast charging station with energy storage

The data is taken form a major supplier of these stations. Slow charging stations also have a limit for the energy since the charging time is limited to avoid overheating.

The **other parameters** used in the case study are listed in Table 1. For the assumptions about the uncertainty, field data is not available to the best knowledge of the authors. That is why no data can be derived for a realistic range of uncertainty and several values for χ and ψ are used which are also listed in the following table.

Table 1 Data for the case study

	Parameter	Value
$\lvert S \rvert$	Number of stops	Deterministic approach: 40 Robust approach: 20
π	Energy charged at terminal	Slow charging station (type 2) for 5 minutes
λ	Number of cycles	10
υ	Minimum energy in the battery	5 kWh
γ	Number of buses	10
ζ	Minimum state of charge	20 %
κ	Maximum state of charge	90 %
$\overline{w}$	Upper bound for battery capacity	40 kWh
χ	Maximum relative deviation	0 %, 10 %, 30 %, 50 %
ψ	Budget of uncertainty in %	0 %, 33 %, 67 %, 100 %

The data about power (ρ_s^t) and energy (ϕ_s^t) limit and the cost (α_s^t) of the stations cannot be published because it is confidential. The same holds for the battery module specific cost β.

Because of limitations of memory for the heuristics that are described later, only $|S| = 20$ stops are considered for the robust aproach.

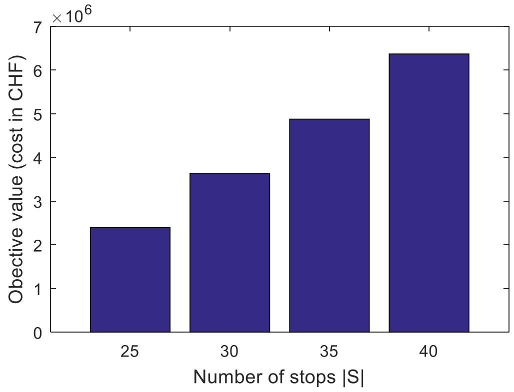
4.2 Computational results

Deterministic model

The overall cost for 40 stops is 6.41 million CHF. With similar data about charging stations, Chen et al. (2013) obtained 4.24 million CHF per year, but they also incorporated electricity cost, drivers' salary and some other cost in their model.

The cost obtained for different numbers of stops is shown in Figure 7. Obviously, additional stops (equal to a longer bus line) increase the cost significantly.

Figure 7 Cost evolution with increasing number of stops (deterministic mixed integer model)

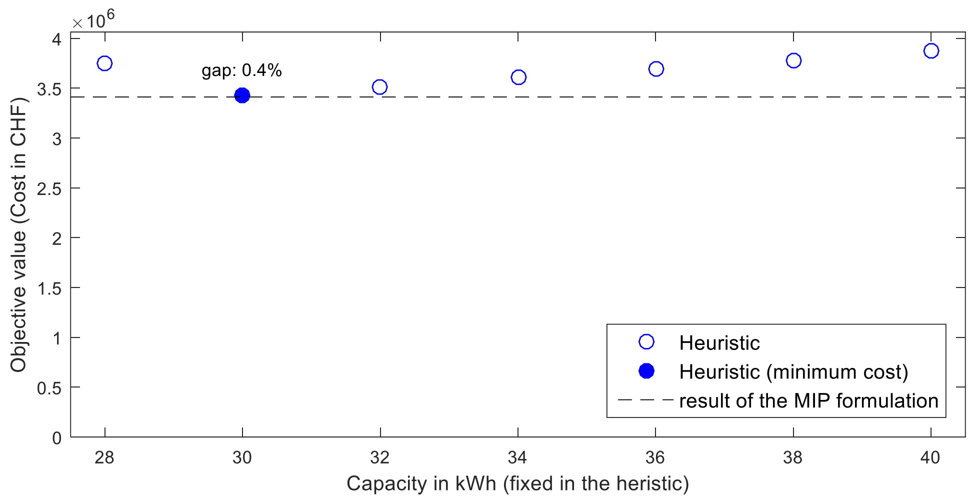


Robust model

For the comparison, the same data as before is used. However, only fast charging stations with energy storage (station type (3)) are considered since the heuristic can only place stations of one type. A line with 20 stops is considered because of memory limitations.

First, the heuristic is tested with zero uncertainty. In this case, the cost difference between the two models only stems from the greedy property of the model and not from the additional cost caused by the uncertainty. Without uncertainty, only one branch is calculated, namely the one where all energy consumptions are the nominal ones. The result is shown in Figure 8 where the objective value of the mixed integer model is shown as a dotted line. The heuristic is calculated for seven pre-defined capacities. Circles in the figure show the heuristic's objective. The capacity with the best objective is marked with a filled circle.

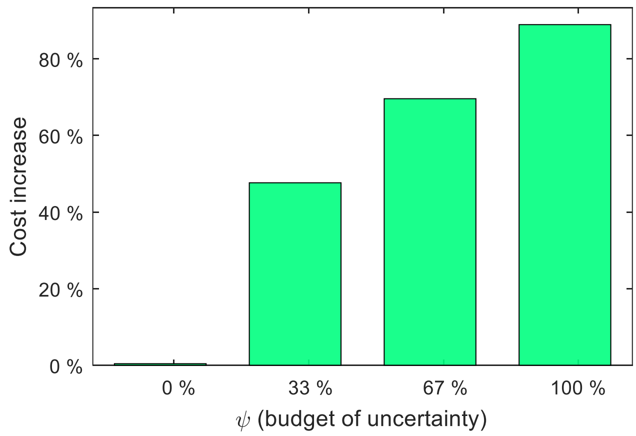
Figure 8 Comparison of mixed integer model (dashed line) and the heuristic if uncertainty is zero. The filled circle shows the minimum cost for the heuristic.



It can be seen that the gap for the capacity with the lowest overall cost is relatively small, but the cost varies much between the capacities. This indicates that the greediness of the heuristic only has a slight impact on the solution quality. Regarding the evolution of cost for different capacities, it is observed that cost increases linearly on the right due to the battery cost that is proportional to battery capacity. On the left, the decrease in cost stems from a reduction of the necessary number of stations.

In the following, uncertainty is integrated into the heuristic. Figure 9 shows the "price of robustness" which is plotted as the cost increase for the different budgets of uncertainty. It is assumed that the energy consumption can be $\chi = 50\%$ higher in some segments.

Figure 9 Gap between robust and deterministic model for $\chi = 50$ %.



Cost increase due to uncertainty is very high. The gaps for all combinations of χ and ψ are shown in the following table.

Table 2 Gap for combinations of χ and ψ

		ψ 0 %	33 %	67 %	100 %
χ	0 %	0.4 %	0.4 %	0.4 %	0.4 %
	10 %	0.4 %	8.2 %	15.3 %	15.3 %
	30 %	0.4 %	27.6 %	39.8 %	52.1 %
	50 %	0.4 %	47.6 %	69.6 %	88.9 %

The first column and the first row correspond to the case without uncertainty. For this case, it can be seen that neither the greedy property of the heuristic nor the suboptimal determination of the battery capacity leads to much higher cost.

However, for the cases without uncertainty, we cannot judge how much the greediness of the heuristic influences the solution quality. On the other hand, by fixing the battery capacity in both the MIP and the heuristic, we can eliminate the suboptimal determination of the capacity. The following table shows the result.

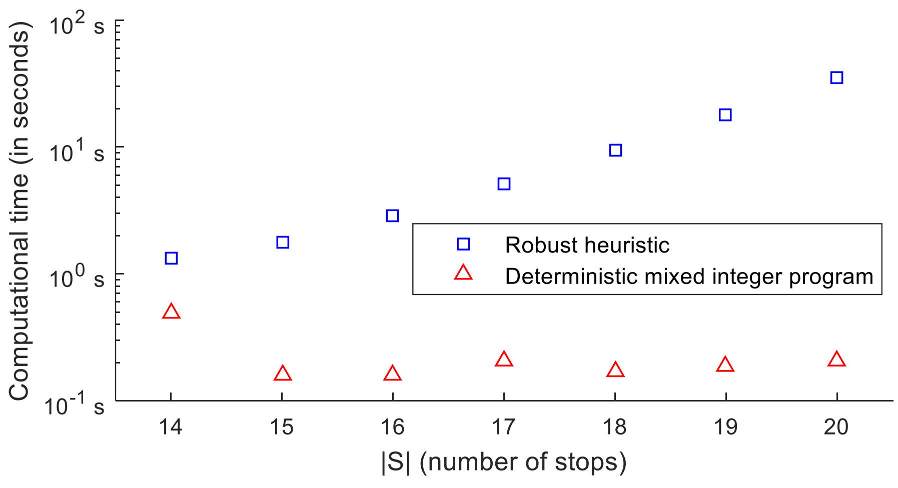
Table 3 Gap for combinations of χ and ψ. Capacity is fixed to 28 kWh.

		ψ			
		0 %	33 %	67 %	100 %
χ	0 %	0.0 %	0.0 %	0.0 %	0.0 %
	10 %	0.0 %	11.2 %	22.3 %	22.3 %
	30 %	0.0 %	33.5 %	44.6 %	44.6 %
	50 %	0.0 %	55.8 %	66.9 %	78.1 %

Without uncertainty, both heuristic and mixed integer program lead to the same number of stations. That leads to the same cost. When it comes to uncertainty, there obviously are some cases where cost increase more than in the upper case and some where the cost decreases. The effect of the suboptimal determination (simply trying out some values) of the capacity can thus not be quantified.

However, there are some issues with the heuristic. The first one is the relatively high computational time that is shown in Figure 10 for different numbers of stops. Each time, the heuristic is calculated for seven pre-defined capacities. To allow comparison, the solution time for the mixed integer model is also plotted.

Figure 10 Comparison of computational time
(The heuristic is calculated for 7 different capacities.)



As expected, the MIP performs much better in terms of computational time than the heuristic since approximately 2^{20} paths need to be calculated. In the calculation, each additional stop makes the computational time to increase by 70 %.

The other issue of the heuristic is related to memory consumption that is similarly growing exponentially. The available memory of 1 GB only allows to calculate a bus line with 22 stops. A line with 40 stops would require more than 200 TB memory in the current implementation.

5 Conclusion and further work

In the first part of this paper, a deterministic model has been presented based on the existing literature about the optimization of bus charging infrastructure. Then, a new heuristic has been developed. The heuristic calculates the energy in the battery at all stops for all possible combinations of high and normal energy consumptions in the segments of the route. During this calculation, a new station is placed when the energy charged at the already existing stations is not sufficient to fulfil the minimum energy requirements at a stop. A greedy algorithm places the stations.

In the case study, the exact deterministic mixed integer model and the heuristic have been tested and compared. The numerical results for the robust heuristic show that the price of uncertainty can be really high and can make the costs to double depending on the assumptions about the uncertainty.

However, further improving the heuristic, e.g. by the use of meta-heuristics, could decrease the cost in the robust model. The problem of memory consumption of the heuristics made them impossible to be applied to problems with more than 22 stops given the used hardware. These issues are to be solved in the further development of the model, mainly by changes in the implementation and by the use of symmetry in the tree.

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