Railway Passenger Service Timetable Design

Tomáš Robenek

Yousef Maknoon

Michel Bierlaire

Transport and Mobility Laboratory, EPFL

April 2015

STRC

15th Swiss Transport Research Conference

Monte Verità / Ascona, April 15 – 17, 2015

Transport and Mobility Laboratory, EPFL

Railway Passenger Service Timetable Design

Tomáš Robenek, Yousef Maknoon, Michel
Bierlaire
Transport and Mobility Laboratory
École Polytechnique Fédérale de Lausanne
Station 18
CH-1015 Lausanne
phone: +41-21-693 24 32
fax: +41-21-693 80 60
{tomas.robenek, yousef.maknoon,
michel.bierlaire}@epfl.ch

April 2015

Abstract

The aim of this paper is to analyze and to improve the current planning process of the passenger railway service. At first, the state-of-the-art in research is presented. Given the recent changes in legislature allowing competitors to enter the railway industry in Europe, also known as liberalization of railways, the current way of planning does not reflect the situation anymore. The original planning is based on the accessibility/mobility concept provided by one carrier, whereas the competitive market consists of several carriers that are driven by the profit.

Moreover, the current practice does not define the ideal timetables (the initial most profitable timetables) and thus it is assumed that the Train Operating Companies (TOCs) use their historical data (train occupation, ticket sales, etc.) in order to construct the ideal timetables.

For the first time in this field, we tackle the problem of ideal timetables in railway industry from the both points of view: TOCs' and passengers'. We propose the Ideal Train Timetabling Problem (ITTP) to create a list of train timetables for each TOC separately. The ITTP approach incorporates the passenger demand in the planning and its aim is to maximize TOCs' profits while keeping the passengers' costs at a certain level. The outcome of the ITTP is the ideal timetables (including connections between the trains), which then serve as inputs for the traditional Train Timetabling Problem (TTP). We test our approach on the S-train network of Canton Vaud, Switzerland.

Keywords

Ideal Railway Timetabling, Cyclic - Non-cyclic Timetable, Mixed Integer Linear Programming

1 Introduction

The time of dominance of one rail operating company (usually the national carrier) over the markets in Europe is reaching to an end. The new EU regulation (EU Directive 91/440) allows open access to the railway infrastructure to companies other than those who own the infrastructure, thus allowing the competition to exist in the market.

Up to this point, the national carriers were subsidized by local governments and their purpose was to offer the accessibility and mobility to the public (passengers). On the other hand, the goal of the private sector is to generate profit, *i.e.* to maximize the captured demand.

However, the passenger demand is subject to the human behavior that incorporates several factors, to list a few: sensitivity to the time of the departure related to the trip purpose (weekday peak hours for work or school, weekends for leisure, *etc.*), comfort, perception and others. Moreover the passenger service has to compete with other transportation modes (car, national air routes, *etc.*) and thus faces even higher pressure to create good quality timetables.

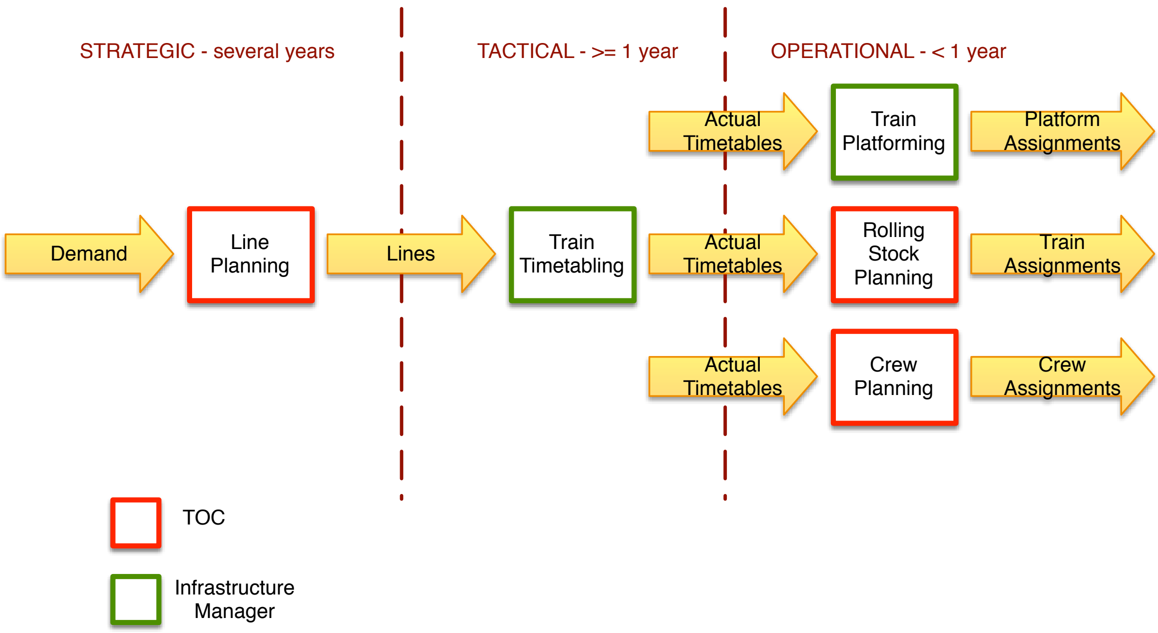

Figure 1: *Planning overview of railway operation*

If we have a closer look at the planning horizon of the railway passenger service (as described in Caprara *et al.* (2007) and visualized on Figure 1), we can see that the issue of ideal departure times has been neglected in the past. The Train Timetabling Problem (TTP) does take as input the ideal timetables (in its non-cyclic version), however the procedure of generating such timetables is missing. Similarly, in the cyclic version of the TTP, the objective function that would maximize the TOC profit or passenger cost (satisfaction) is undefined.

We believe that the lack of the definition of the ideal timetables and how to create them, is a major gap, caused by the lack of a competition in the previous railway market settings. We assume that the lack of the passenger input in the planning, lead to the decrease of the railway mode share in the transportation market.

And thus we propose to insert an additional section in the planning horizon called the Ideal Train Timetabling Problem (ITTP). In the ITTP, we introduce a definition of the ideal timetable as follows: the ideal timetable, consists of train departures, such that the profit associated with each scheduled train is maximal while the passenger cost associated with the timetable is within predefined limit. Such a timetable would benefit both, passengers and the TOC(s) in the respective manner: the TOC would set acceptable level of passenger costs, which would turn forecasted demand into the realized demand and the TOC's profit, by definition, would be maximized.

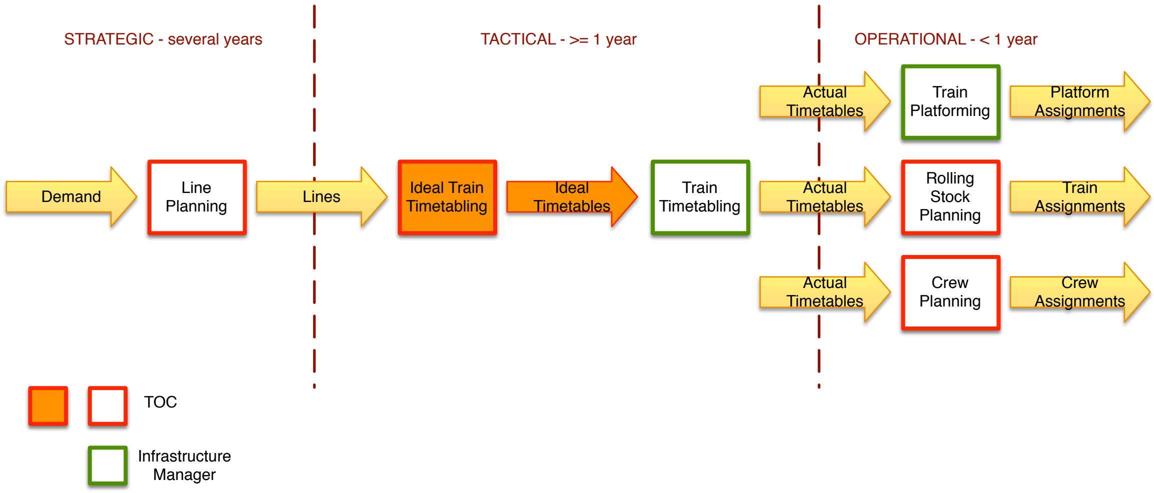

Figure 2: *Modified overview of railway operation*

The ITTP is using the output of the Line Planning Problem (LPP) and serves as an input to the traditional TTP and hence, it is placed between the two respective problems (Figure 2). The driver of this problem is the profit function of a TOC. The model will allow timetables of the TOC's train lines to take the form of the non-cyclic or cyclic schedule. Moreover, we introduce a demand induced connections. The connections between the trains are not pre-defined, but are subject to the demand (via passengers' costs). In the literature the connections are handled only in the cyclic version of the TTP, where they are always induced, without a proper reasoning.

The structure of the manuscript is as follows: the literature review of the state-of-the-art (Section 2) is followed by a formal definition of a passenger cost (Section 3), leading to a problem definition and its mathematical formulation (Section 4). The model is tested on a Swiss case study (Section 5). The paper is finalized by drawing some conclusions and discussion of possible extensions (Section 6).

2 Literature Review

The state-of-the-art literature is mostly focused on the traditional planning problems and considers the passengers (in the form of hourly demand) only in the initial phase (*i.e.* the LPP). Due to the extensiveness of the literature, we focus on reviewing of the classical TTP as the ITTP's goal is to provide better information for the TTP using the outcome of the LPP.

The aim of the TTP is to find a feasible (operational) timetable for a whole railway network, *i.e.* there are no conflicts of the trains using the tracks. In the non-cyclic version, ideal timetables with their respective profits serve as the main input. The TTP then shifts the departures for conflicting trains, such that the losses of the profits are minimized. In the cyclic version, the model searches for a first feasible timetable given the size of the cycle. The user can create his/her own objective function, otherwise arbitrary solution will be selected.

2.1 Non-Cyclic TTP

Most of the models, on the non-cyclic timetabling, in the published literature, formulate the problem either as Mixed Integer Linear Programming (MILP) or Integer Linear Programming (ILP). The MILP model uses continuous time, whereas the ILP model discretizes the time. Due to the complexity of the problem, many heuristic approaches are considered.

One of the first TTP papers is Brannlund *et al.* (1998). The authors discretize the time and solve the problem using lagrangian relaxation of the track capacity constraints, *i.e.* the model is formulated as an ILP. The lagrangian relaxation of the same constraints is used as well in Caprara *et al.* (2002, 2006), Fischer *et al.* (2008) and Cacchiani *et al.* (2012). On the other hand, in Cacchiani *et al.* (2008), column generation approach is tested. This approach tends to find better bounds than the lagrangian relaxation. Subsequently, several ILP re-formulations are introduced and compared in Cacchiani *et al.* (2010a). In Cacchiani *et al.* (2010b), the ILP formulation is adjusted, in order to be able to schedule extra freight trains, whilst keeping the timetables of the passengers' trains fixed. The dynamic programming approach, to solve the clique constraints, is used in Cacchiani *et al.* (2013).

The MILP model has received less attention in the literature. In Carey and Lockwood (1995), a heuristic, that considers one train at a time and solves the MILP, based on the already scheduled trains, is introduced. Several more heuristics to solve the MILP model are presented in Higgins *et al.* (1997).

A few different models exist: Oliveira and Smith (2000) and Burdett and Kozan (2010), reformulate the problem as a job-shop scheduling one. Erol (2009), Caprara (2010) and Harrod (2012), survey different types of models for the TTP.

None of the above formally defines the ideal timetable. The models focus on the feasibility of the solutions, *i.e.* the track occupation constraints. Demand is omitted in the formulations.

2.2 Cyclic TTP

One of the first papers, dealing with cyclic timetables is Serafini and Ukovich (1989). The paper brings up the topic of cyclic scheduling based on the Periodic Event Scheduling Problem (PESP). The problem is solved with an algorithm using implicit enumeration and network flow theory. In Nachtigall and Voget (1996) model for minimization of the waiting times in the railway network, whilst keeping the cyclic timetables (based on PESP), is solved using branch and bound. The same model is solved using genetic algorithms in Nachtigall (1996). The general PESP model is solved using constraint generation algorithm in Odijk (1996) and with branch and bound in Lindner and Zimmermann (2000).

In Kroon and Peeters (2003), variable trip times are considered. Peeters (2003) then further elaborates on PESP and discusses different forms of the objective function. In Liebchen and Mohring (2002), the PESP attributes are analyzed on the case study of Berlin's underground and in Liebchen (2004) implementation of the symmetry in the PESP model is presented. Lindner and Zimmermann (2005) propose to use decomposition based branch and bound algorithm to solve the PESP.

Kroon *et al.* (2007) and Shafia *et al.* (2012), deal with robustness of cyclic timetables. Liebchen and Mohring (2004) propose to integrate network planning, line planning and rolling stock scheduling into the one periodic timetabling model (based on PESP). Similarly, Kaspi and Raviv (2013) propose to integrate TTP with LPP. The new objective function is minimizing the passengers' total travel time and the TOC's operating cost (in terms of operating time).

Caimi *et al.* (2007) and Kroon *et al.* (2014) introduce flexible PESP – instead of the fixed times of the events, time windows are provided. Lastly, Chierici *et al.* (2004) is maximizing the captured demand using demand estimation (logit model).

3 Passenger Cost

In order to find a good timetable from the passenger point of view, we need to take into account passenger behavior. Such a behavior can be modeled using discrete choice theory (Ben-Akiva and Lerman (1985)). The base assumption in discrete choice theory is that the passengers maximize their utility, *i.e.* minimize the cost associated with each alternative and select the best one.

We propose the following costs associated with passengers' ideal timetable:

- in-vehicle-time (VT)
- waiting time (WT)
- number of transfers (NT)
- scheduled delay (SD)

The **in-vehicle-time** is the (total) time passengers spend on board of (each) train. This time allows the passengers to distinguish between the "slow" and the "fast" services.

The **waiting time** is the time passengers spend waiting between two consecutive trains in their respective transfer points. The cost perception related to the waiting time is evaluated as double and a half of the in-vehicle-time (see Wardman (2004)).

The **transfer(s)** aim at distinguishing between direct and interchange services. In literature and practice, it is by adding extra travel (in-vehicle) time to the overall journey. In our case, we have followed the example of Dutch Railways (NS), where penalty of 10 minutes per transfer is applied (see de Keizer *et al.* (2012)). Even though variety of studies show that number of interchanges, distance walked, weather, *etc.* play effect in the process, it is rather difficult to incorporate in optimization models. Thus using the applied value (by NS) will bring this research closer to the industry.

The **scheduled delay** is indicating the time of the day passengers want to travel, *i.e.* following the assumption that the demand is time dependent. For example: most of the people have to be at their workplace at 8 a.m. Since it is impossible to provide service that would secure ideal arrival time to the destination for everyone, scheduled delay functions are applied (Figure 3).

As shown in Small (1982), the passengers are willing to shift their arrival time by 1 to 2 minutes earlier, if it will save them 1 minute of the in-vehicle-time, similarly they would shift their arrival by 1/3 to 1 minute later for the same in-vehicle-time saving. If we would consider the boundary case, the lateness ($f_1 = 1$) is perceived equal to the in-vehicle-time and earliness ($f_2 = 0.5$) has

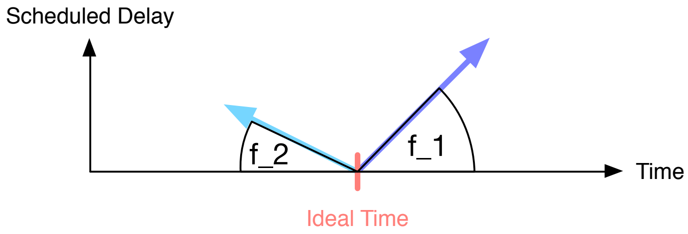

Figure 3: *Scheduled Delay Functions*

half of the value (as seen on Figure 3).

To estimate the perceived cost (quality) of the selected itinerary in a given timetable for a single passenger, we sum up all the characteristics:

$$C = VT + 2.5 \cdot WT + 10 \cdot NT + SD\,[\text{min}] \tag{1}$$

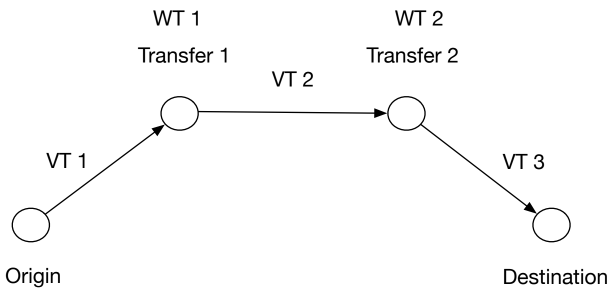

Figure 4: *Example Network*

For a better understanding, consider the following example using network on Figure 4: passenger's itinerary consists of taking 3 consecutive trains in order to go from his origin to his destination, he has to change train twice. If he arrives to his destination earlier than his ideal time, his SD will be:

$$SD_e = \text{argmax}\left(\frac{ideal\ time - arrival\ time}{2}\ ,\ 0\right) \tag{2}$$

We use argmax function as one train line has several trains per day scheduled and the passenger selects the one closest to his desired traveling time. On the other hand, if he arrives later than his ideal time, then his SD will be:

$$SD_l = \text{argmax}\,(0\,,\ \textit{arrival time} - \textit{ideal time}) \tag{3}$$

The overall scheduled delay is then formed:

$$SD = \text{argmin}\,(SD_e\,,\ SD_l) \tag{4}$$

His overall perceived cost will be the following:

$$C = \sum_{trains} VT + 2.5 \cdot \sum_{transfers} WT + 10 \cdot NT + SD\,[\text{min}] \tag{5}$$

The resulting value is in minutes, however it is often desirable to estimate the cost in monetary values for pricing purposes. In such a case, national surveys estimating respective nation's value of time (VOT) exist. The VOT is given in nation's currency per hour, for instance in Switzerland the VOT for commuters using public transport is 27.81 swiss francs per hour (Axhausen *et al.* (2008)). To make the cost in monetary units, simply multiply the whole Equation 1 by the VOT/60.

The aim of our research is not to calibrate the weights in Equation 1, but to provide better timetables in terms of the departure times. The weights serve as an input for our problem and thus can be changed at any time. Adding everything up, the ideal timetable from the passenger point of view can be defined as follows:

> The ideal timetable consists of train departure times that passengers' global costs are minimized, *i.e.* the most convenient path to go from an origin to a destination traded-off by a timely arrival to the destination for every passenger.

Similar concept, improving quality of timetables has been done in Vansteenwegen and Oudheusden (2006, 2007). Their approach has been focused on reliable connections for transferring passengers, whereas in our framework we focus on the overall satisfaction of every passenger.

Other concept similar to ours has been used in the delay management, namely in Kanai *et al.* (2011) and Sato *et al.* (2013). However their definition of dissatisfaction of passengers omits the scheduled delay.

4 Mathematical Formulation

In this section, we present a mixed integer programming formulation for the Ideal Train Timetabling Problem.

The aim of this problem is to define and to provide the ideal timetables as input for the traditional TTP. It is not well said in the TTP, what ideal means. It is only briefly mentioned, that supposedly, those are the timetables, that bring the most profit to the TOCs (this assumption is in line with the competitive market). Generally speaking, the more of the demand captured, the higher the profit. Since the demand estimation is a complex task that requires a lot of data collection, in this manuscript, we propose to use a linear estimate of a passenger cost (as described in the previous section). If this cost is minimized, we can assume that a passenger would choose the train as his/her mode of transport. However, this approach leads to a multi-objective problem (profit versus cost). In order to avoid this issue, we propose to treat the passenger cost as an ϵ-constraint (**?**). Thus the ITTP's goal is to design TOC's timetables, such that its profit is maximized whilst securing an ϵ level of the total passenger cost.

The input of the ITTP is the demand that takes the form of the amount of passengers that want to travel between OD pair $i \in I$ and that want to arrive to their destination at their ideal time $t \in T_i$. Apart of that, there is a pool of lines $l \in L$ and its segments $g \in G^l$. Segment is a part of the line between two stations, where the train does not stop. Each line has an assigned frequency expressed as the available trains $v \in V^l$ (lines, segments and frequencies are the output of the LPP). Based on the pool of lines, the set of paths between every OD pair $p \in P_i$ can be generated. The path is called an ordered sequence of lines to get from an origin to a destination including details such as the running time from the origin of the line to the origin of the OD pair h_i^{pl} (where $l = 1$), the running time from an origin of the OD pair to a transferring point between two lines r_i^{pl} (where $l = 1$), the running time from the origin of the line to the transferring point in the path h_i^{pl} (where $l > 1$ and $l < |L^p|$), the running time from one transferring point to another r_i^{pl} (where $l > 1$ and $l < |L^p|$) and the running time from the last transferring point to a destination of the OD pair r_i^{pl} (where $l = |L^p|$). Note that the index p is always present as different lines using the same track might have different running times.

Part of the ITTP is the routing of the passengers through the railway network. Using a decision variable x_i^{tp}, we secure that each passenger (combination of indices *it*) can use exactly one path. Similarly, within the path, passenger can use exactly one train on every line in the path (decision variable y_i^{tplv}). These decision variables, among others, allow us to backtrace the exact itinerary of every passenger. The timetable is understood as a set of departures for every train on every line (values of d_v^l). The timetable can take form of a non-cyclic or a cyclic version (depending if the cyclicity constraints are active, see below).

Since we know the exact itinerary of every passenger, we can measure the train occupation o_{vg}^l of every train v of every line l on each of its segment g. Derived from the occupation, number of train units u_v^l is assigned to each train. This value can be equal to zero, which means that the train is not running and the frequency of the line can be reduced. The model also keeps track of the number of train drivers α_v^l needed to realize the timetable.

We can formulate the ITTP as follows:

Sets Following is the list of sets used in the model:

I	–	set of origin-destination pairs
T_i	–	set of ideal times for OD pair i
P_i	–	set of possible paths between OD pair i
L	–	set of operated lines
L^p	–	set of lines in the path p
V^l	–	set of available trains for the line l (frequency)
G^l	–	set of segments on line l

Input Parameters Following is the list of parameters used in the model:

M	–	sufficiently large number (for daily planning in minutes, the value can be 1440)
m	–	minimum transfer time[min]
c	–	cycle[min]
r_i^{pl}	–	running time between OD pair i on path p using line l[min]
h_i^{pl}	–	time to arrive from the starting station of the line l to the origin/transferring point of the OD pair i in the path p[min]

D_i^t – demand between OD pair i with ideal time t[passengers]

q_t – value of the in vehicle time [monetary units per minute]

q_w – value of the waiting time in the relation to the VOT [unitless]

f_1 – coefficient of being early in the relation to the VOT [unitless]

f_2 – coefficient of being late in the relation to the VOT [unitless]

a – penalty for having a train transfer [min]

b_g – ticket price of a segment g[monetary units]

e – cost of a train driver [monetary units/train-km]

n – operating cost of a single train unit [monetary units/km]

β – capacity of a single train unit [passengers]

j – maximum length of the train [train units]

k^l – length of the line l [km]

ϵ – maximum total passenger cost allowed [monetary units]

Decision Variables Following is the list of decision variables used in the model:

C_i^t – the total cost of a passenger with ideal time t between OD pair i

w_i^t – the total waiting time of a passenger with ideal time t between OD pair i

w_i^{tp} – the total waiting time of a passenger with ideal time t between OD pair i using path p

w_i^{tpl} – the waiting time of a passenger with ideal time t between OD pair i on the line l that is part of the path p, i.e. the waiting time in the transferring point, when transferring to line l

x_i^{tp} – 1 – if passenger with ideal time t between OD pair i chooses path p; 0 – otherwise

s_i^t – the value of the scheduled delay of a passenger with ideal time t between OD pair i

s_i^{tp} – the value of the scheduled delay of a passenger with ideal time t between OD pair i traveling on the path p

d^l_v – the departure time of a train v on the line l (from its first station)

y^{tplv}_i – 1 – if a passenger with ideal time t between OD pair i on the path p takes the train v on the line l; 0 – otherwise

z^l_v – dummy variable to help modeling the cyclicity corresponding to a train v on the line l

o^l_{vg} – train occupation of a train v of the line l on a segment g

u^l_v – number of train units of a train v on the line l

α^l_v – 1 – if a train v on the line l is being operated; 0 – otherwise

Routing Model The ITTP model can be decomposed into 2 parts: routing and pricing. The routing takes care of the feasibility of the solution, whereas pricing takes care of the passenger cost attributes. At first, we present the routing of the passengers – the Routing Model (RM):

$$\max \sum_{l\in L}\sum_{v\in V^l}\sum_{g\in G^l} o^l_{vg}\cdot b_g - \sum_{l\in L}\sum_{v\in V^l}\left(\alpha^l_v\cdot e\cdot k^l + u^l_v\cdot n\cdot k^l\right) \tag{6}$$

$$\sum_{i\in I}\sum_{t\in T_i} D^t_i\cdot C^t_i \le \epsilon, \tag{7}$$

$$\sum_{p\in P_i} x^{tp}_i = 1, \qquad \forall i\in I, \forall t\in T_i, \tag{8}$$

$$\sum_{v\in V^l} y^{tplv}_i = 1, \qquad \forall i\in I, \forall t\in T_i, \forall p\in P_i, \forall l\in L^p, \tag{9}$$

$$\left(d^l_v - d^l_{v-1}\right) = c\cdot z^l_v, \qquad \forall l\in L, \forall v\in V^l : v>1, \tag{10}$$

$$o^l_{vg} = \sum_{i\in I}\sum_{t\in T_i}\sum_{p\in P_i} y^{tplv}_i\cdot D^t_i, \qquad \forall l\in L, \forall v\in V^l, \forall g\in G^l, \tag{11}$$

$$u^l_v\cdot\beta \ge o^l_{vg}, \qquad \forall l\in L, \forall v\in V^l, \forall g\in G^l, \tag{12}$$

$$\alpha^l_v\cdot j \ge u^l_v, \qquad \forall l\in L, \forall v\in V^l, \tag{13}$$

$$C^t_i \ge 0, \qquad \forall i\in I, \forall t\in T_i, \tag{14}$$

$$d^l_v \ge 0, \qquad \forall l\in L, \forall v\in V^l, \tag{15}$$

$$x^{tp}_i \in (0,1), \qquad \forall i\in I, \forall t\in T_i, \forall p\in P_i, \tag{16}$$

$$y^{tplv}_i \in (0,1), \qquad \forall i\in I, \forall t\in T_i, \forall p\in P_i, \forall l\in L^p, \forall v\in V^l, \tag{17}$$

$$o^l_{vg} \ge 0, \qquad \forall l\in L, \forall v\in V^l, \forall g\in G^l, \tag{18}$$

$$u^l_v \in (0,j), \qquad \forall l\in L, \forall v\in V^l, \tag{19}$$

$$\alpha^l_v \in (0,1), \qquad \forall l\in L, \forall v\in V^l, \tag{20}$$

$$z^l_v \in \mathbb{N}, \qquad \forall l\in L, \forall v\in V^l. \tag{21}$$

The objective function **(6)** aims at maximizing the TOC's profit. It consists of two terms: revenue minus operating cost. The constraints **(7)** assure that a certain level of the total passenger cost (ϵ) will be maintained. Constraints **(8)** secure that every passenger is using exactly one path to get from his/her origin to his/her destination. Similarly constraints **(9)** make sure that every passenger takes exactly one train on each of the lines in his/her path. Constraints **(10)** model the cyclicity using integer division. When solving the non-cyclic version of the problem, these constraints have to be removed. Constraints **(11)** keep track of a train occupation. Constraints **(12)** verify that the train capacity is not exceeded on every stretch/segment of the line. Constraints **(13)** assign train drivers, *i.e.* if a train v on the line l is being operated or not. Constraints **(14)–(21)** set the domains of decision variables.

Pricing Constraints To make the ITTP complete, we need to expand the Routing Model with the pricing constraints. We will add the pricing constraints in blocks of attributes that create the cost of a passenger.

$$
\begin{aligned}
s_i^t \geq s_i^{tp} - M \cdot \left(1 - x_i^{tp}\right), \qquad & \forall i \in I, \forall t \in T_i, \forall p \in P_i, && (22)\\
s_i^{tp} \geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t\right) & \\
-M \cdot \left(1 - y_i^{tp|L|v}\right), \qquad & \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall v \in V^{|L|}, && (23)\\
s_i^{tp} \geq f_1 \cdot \left(t - \left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right)\right) & \\
-M \cdot \left(1 - y_i^{tp|L|v}\right), \qquad & \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall v \in V^{|L|}, && (24)\\
s_i^t \geq 0, \qquad & \forall i \in I, \forall t \in T_i, && (25)\\
s_i^{tp} \geq 0, \qquad & \forall i \in I, \forall t \in T_i, \forall p \in P_i. && (26)
\end{aligned}
$$

The first block of constraints takes care of the **scheduled delay (SD)**. In our model we have 2 types of scheduled delay: SD for every path (constraints **(26)**) and SD that is linked to the path, which will be the final selected path of a given passenger(s) with a given ideal time (constraints **(25)**). As described in the Section 3, the constraints **(23)** model the earliness of the passengers (Equation **2**) and constraints **(24)** model the lateness (Equation **3**). Constraints **(22)** make sure that only one SD is selected (Equation **4**) – not necessarily the lowest one as it depends on the cost of the whole itinerary (constraints **(34)**), *i.e.* the path with the smallest overall cost will be selected for the given OD pair with a given ideal time. These constraints also allow us to avoid the non-linearity in the estimation of the final passenger cost (constraints **(34)**).

$$w_i^t \geq w_i^{tp} - M \cdot \left(1 - x_i^{tp}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{27}$$

$$w_i^{tp} = \sum_{l \in L^p \setminus 1} w_i^{tpl}, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{28}$$

$$\begin{aligned} w_i^{tpl} \geq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \qquad & \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p : \\ -M \cdot \left(1 - y_i^{tpl'v'}\right) - M \cdot \left(1 - y_i^{tplv}\right), \qquad & l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \end{aligned} \tag{29}$$

$$\begin{aligned} w_i^{tpl} \leq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \qquad & \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p : \\ +M \cdot \left(1 - y_i^{tpl'v'}\right) + M \cdot \left(1 - y_i^{tplv}\right), \qquad & l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \end{aligned} \tag{30}$$

$$w_i^t \geq 0, \qquad \forall i \in I, \forall t \in T_i, \tag{31}$$

$$w_i^{tp} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{32}$$

$$w_i^{tpl} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p. \tag{33}$$

The second block of constraints is modeling the **waiting time (WD)**. There are 3 types of waiting time: the final selected waiting time in the best path (constraints **(31)**), the total waiting time of every path (constraints **(32)**) and the waiting time at every transferring point in every path (constraints **(33)**). The constraints **(29)** and **(30)** are complementary constraints that model the waiting time in the transferring points in every path. In other words, these two constraints find the two best connected trains in the two train lines in the passengers' path. Constraints **(28)** add up all the waiting times in one path to estimate the total waiting time in a given path. Constraints **(27)** make sure that only one WT is selected (similarly as constraints **(22)** for SD).

$$\begin{aligned} C_i^t = q_v \cdot q_w \cdot w_i^t + q_v \cdot a \cdot \sum_{p \in P} x_i^{tp} \cdot (|L^p| - 1) \\ + q_v \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{tp} + q_v \cdot s_i^t, \qquad \forall i \in I, \forall t \in T_i. \end{aligned} \tag{34}$$

At last, constraints **(34)** combine all the attributes together as in Equation **5** multiplied by the VOT. The complete ITTP model can be seen in Appendix 8.

5 Case Study

In order to test the ITTP model, we have selected the network of S-trains in canton Vaud, Switzerland as our case study. The reduced network is represented on Figure 5 (as of timetable 2014). We consider only the main stations in the network (in total 13 stations). A simple algorithm in Java has been coded, in order to find all the possible paths between every OD pair. The algorithm allowed maximum of 3 consecutive lines to get from an origin to a destination. The traveling times have been extracted from the Swiss Federal Railways' (SBB) website. The minimum transfer time between two trains has been set to 4 minutes. The cycle in Switzerland is one hour.

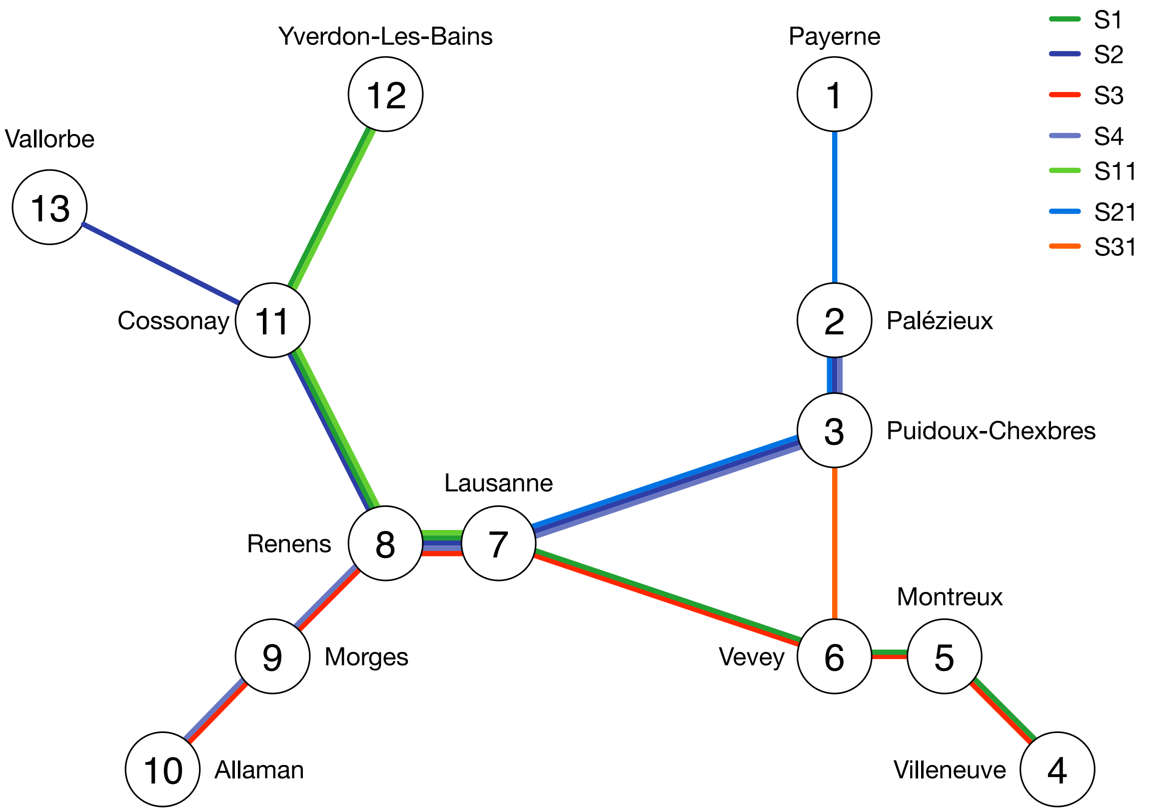

Figure 5: *Network of S-trains in canton Vaud, Switzerland*

In Table 1, you can find the list of all S-train lines of the canton Vaud in the timetable of 2014. There are 7 lines that run in both directions. Each combination of a line and its direction has its unique ID number. Column "from" marks the origin station of the line as well as column "to" marks its destination. The columns "departures" show the currently operated timetable in the morning peak hour (5 a.m. to 9 a.m.), which is the time horizon used in our study. Trains that did not follow the cycle (marked with a star *) were set to a cycle value, in order to not violate the cyclicity constraints (the timetables in Switzerland are cyclic with a cycle of one hour).

Line	ID	From	To	Departures			
S1	1	Yverdon-les-Bains	Villeneuve	–	6:19	7:19	8:19
	2	Villeneuve	Yverdon-les-Bains	5:24	6:24	7:24	8:24
S2	3	Vallorbe	Palézieux	5:43	6:43	7:43	8:43
	4	Palézieux	Vallorbe	–	6:08	7:08	8:08
S3	5	Allaman	Villeneuve	–	6:08	7:08	8:08
	6	Villeneuve	Allaman	–	6:53	7:53	8:53
S4	7	Allaman	Palézieux	5:41	6:41	7:41	8:41
	8	Palézieux	Allaman	–	6:35	7:35	8:35
S11	9	Yverdon-les-Bains	Lausanne	5:26*	6:34	7:34	8:34
	10	Lausanne	Yverdon-les-Bains	5:55	6:55	7:55	8:55
S21	11	Payerne	Lausanne	5:39	6:39	7:38*	8:39
	12	Lausanne	Payerne	5:24	6:24	7:24	8:24
S31	13	Vevey	Puidoux-Chexbres	–	6:09	7:09	8:09
	14	Puidoux-Chexbres	Vevey	–	6:31*	7:36	8:36

Table 1: *List of S-train lines in canton Vaud, Switzerland*

The SBB is operating the Stadler Flirt train units on the lines S1, S2, S3 and S4. In our case study, we have homogenized the fleet and thus use this type of a train also for the rest of the lines. The capacity of this unit is 160 seats and 220 standing people. The operating cost has been taken from the SBB's annual report for 2013 (**?**), where a regional service has a cost of 30 CHF of a train per kilometer. Since no further details have been provided, we had to break down the cost using external knowledge. From a project for a swiss public transport operator, we know that the driver cost is more than a half of the cost, *i.e.* in the "worst" case it is equal to a half (the higher the cost of the driver the cheaper the operating cost of additional train units thus the worst case). This leads to an operating cost of 15 CHF per train unit per km. The length of the lines in kilometers has been estimated using Google Maps. The maximum amount of train units per train is 2 (as SBB never uses more units). The amount of train units per train remains the same along the line, but it might change at the end stations (we don't go into further details as this is the task of the Rolling Stock Problem).

The ticket prices have been taken directly from the SBB website. In Switzerland, many people have so called General Abonnement (GA) or a half-fare card. With GA, you pay yearly fee and get a free access to almost all public transportation in Switzerland. Half-fare card also comprises of a yearly fee (significantly smaller than GA) and gives access to a half price tickets for public transportation. In our case study, we have applied the half-fare prices to all of the passengers. This approach will balance the prices between GA users and normal users (normal user does not posses GA or half-fare card and thus pays the full price).

The total amount of passengers in the network has been estimated based on its bottleneck (track between Vevey and Puidoux-Chexbres is served only by the line S31, which has the smallest amount of trains scheduled): frequency (in both directions) times maximum train units times a single train unit capacity = 6 times 2 times 380 = 4560. We consider both directions as the OD pairs are generated randomly (uniformed distribution), *i.e.* in this case, the probability of each direction is 50 percent. In order to ease the size of the generated lp file, the passengers have been split into groups of size varying between 1 and 8 (uniform distribution), leaving us with 1000 passenger groups (indices *it*). These groups have been divided into hourly rates (Figure 6) according to the SBB report (**?**) and smoothed into minutes using non-homogenous Poisson process. Since we use concept of an ideal arrival time to the destination, the generated arrival time at the origin has been shifted, by adding up the shortest path travel time between the OD pair, to the destination of the passengers. In total there are 4465 passengers in the network (the deviation from the maximal value of 4560 is due to the randomness).

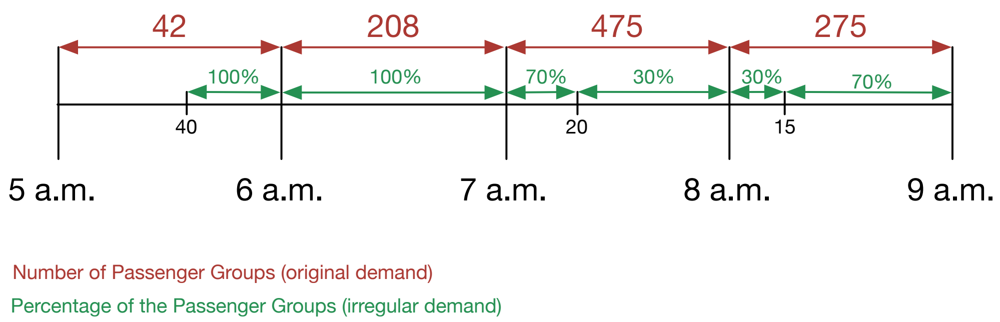

Figure 6: *The hourly distribution of the passenger groups*

The coefficients of the passenger cost are as described in Section 3. The list of all parameter values can be found in Appendix 9

5.1 Technical Details

All of the tested instances have been generated as lp files using Java language and then run in CPLEX Interactive Optimizer (CPLEX version 12.5.1) on a Unix server with 8 cores of 3.33 GHz and 62 GiB RAM. The CPLEX time limit has been set to 2 hours. The goal of this study is the verification of the model rather than the speed of the solution method and the optimality of the solutions. We report bounds for all of the instances and we would like to highlight that due to the many big M constraints it is difficult for CPLEX to close the gap.

In order to strengthen the ITTP formulation, we suggest to add the following constraint:

$$d_v^l \leq d_{v+1}^l - 1, \qquad \forall l \in L, \forall v \in V : v < |V|. \tag{35}$$

this constraint helps to reduce the amount of possible combinations of the values of the train departure times and it is suitable for both cyclic and (especially) non-cyclic formulation.

For this case study, we can further modify the cyclicity constraints to the following form:

$$d_v^l - d_{v-1}^l = 60 \cdot z_{v1}^l + 120 \cdot z_{v2}^l, \qquad \forall l \in L, \forall v \in V : v > 1, \tag{36}$$

$$z_{v1}^l + z_{v2}^l = 1, \qquad \forall l \in L, \forall v \in V : v > 1. \tag{37}$$

As we can notice in the Table 1, we either have 4 trains over 4 hours horizon (60 minutes difference between all consecutive trains) or 3 trains over 4 hours horizon (one train will have 120 minutes time distance from the next train, whereas the other trains will keep the 60 minutes difference). This attribute is modeled by adding an extra index to the cyclicity variable z, stating if the difference between two consecutive trains is 60 or 120 minutes.

In all of the experiments, we have run 3 types of the ITTP model: current, cyclic and non-cyclic. The current model reflects the currently operated SBB timetable as in Table 1 (the decision variables d have been set to the values in the table). The cyclic model does not have the departure times as a hard constraint and thus the CPLEX can look for better values than those of the SBB. The non-cyclic model differs from the cyclic one by removing the cyclicity constraints. In order to speed up CPLEX, we would first solve the current version and give its solution as a warm start for the cylic model and solve it. Subsequentely, we would give the solution of the cyclic model as a warm start to the non-cyclic model.

5.2 Results

In this section, we present the results of our case study. The detailed numerical results can be found in the Tables 2, 3 and 4. The first row of the tables represents the level of the passenger cost in percentage with respect to its base value (100 percent is when we omit the profit and minimize the cost, 0 percent is when we maximize the profit without caring about the cost).

Subsequently, the percentages in between represent the gap between the best and the worst cost, *i.e.* the value of 40 percent has been estimated as (290 094 minus 148 197) times (40 divided by 100). The actual values of the ϵ have been kept same for all three models even though the best cost for cyclic and non-cyclic models were better. This allows us to directly compare the profits at the given ϵ levels and to feed warm starts to the subsequent models.

ϵ [%]	0	20	40	60	80	100
profit [CHF]	175 185	175 180	175 108	172 630	155 554	146 099
cost [CHF]	290 094	261 713	233 334	204 955	176 576	148 197
lb [CHF]	175 711	175 711	175 711	175 711	175 711	132 489
gap [%]	0.30	0.30	0.34	1.78	12.96	10.60
gap [CHF]	526	531	603	3 081	20 157	15 708
time [s]	7 200	7 200	7 200	7 200	7 200	7 200
drivers [-]	36	36	36	39	47	48
rolling stock [-]	64	64	64	65	79	84

Table 2: *Computational results of the current model*

ϵ [%]	0	20	40	60	80	100
profit [CHF]	175 185	175 180	175 108	172 630	155 554	144 492
cost [CHF]	290 094	261 713	233 334	204 955	176 576	138 140
lb [CHF]	176 543	176 543	176 543	176 543	176 543	99 153
gap [%]	0.78	0.78	0.82	2.27	13.49	28.22
gap [CHF]	1 358	1 363	1 435	3 913	20 989	38 987
time [s]	7 200	7 200	7 200	7 200	7 200	7 200
drivers [-]	36	36	36	39	47	48
rolling stock [-]	64	64	64	65	79	87

Table 3: *Computational results of the cylic model*

The three models yield the same results for the ϵ between 0 and 60 percent. This leads to a conclusion that with the smaller amount of trains being scheduled, the passengers have less options and the cyclicity has no influence. Indeed, when the frequency of a line is smaller than the horizon divided by the cycle, the timetable could be perceived by the passengers as non-cyclic. The maximum profit is the same due to the fact that we serve the same passengers in all three cases and thus the minimum amount of trains needed to serve them is the same.The current and the cyclic model also yield the same results for the ϵ of 80 percent, but with a gap around 13 percent. The difference, between the current and the cyclic model, comes at the ϵ of 100 percent, where 3 extra train units of rolling stock are being used and the departure times for

ϵ [%]	0	20	40	60	80	100
profit [CHF]	175 185	175 180	175 108	172 630	155 590	144 971
cost [CHF]	290 094	261 713	233 334	204 955	176 576	135 455
lb [CHF]	176 543	176 543	176 543	176 543	176 543	97 706
gap [%]	0.78	0.78	0.82	2.27	13.47	27.87
gap [CHF]	1 358	1 363	1 435	3 913	20 953	37 749
time [s]	7 200	7 200	7 200	7 200	7 200	7 200
drivers [-]	36	36	36	39	47	47
rolling stock [-]	64	64	64	65	79	85

Table 4: *Computational results of the non-cylic model*

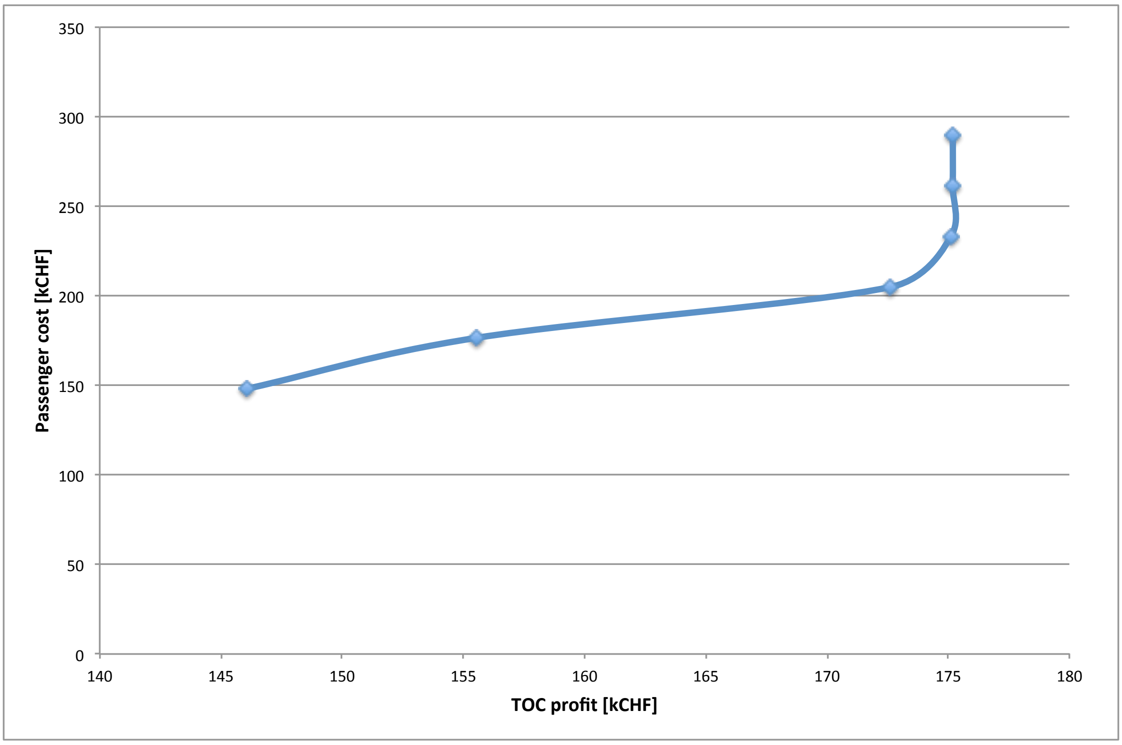

Figure 7: *The Pareto frontier of the current model*

8 out of the 14 lines are slightly shifted. This leads to improvement of cca. 10 000 CHF for the passengers and the deacrese of the TOC's profit by cca. 1 500 CHF.

For the non-cyclic model, the difference starts already at the ϵ of 80 percent, where the TOC profit was slightly improved by 36 CHF. The relative difference at the ϵ 100 percent between the cyclic and the non-cyclic model is rather small – roughly 2 500 CHF better passenger cost and 500 CHF better TOC's profit (less trains scheduled, less train units used). On this results, we can conclude that it is indeed better to use the cyclic timetables over the non-cyclic timetables.

Lastly, we plot the trade-off between the profit and the cost as a Pareto frontier (Figure 7) for the current model. From this figure, we can read that if the TOC is willing to decrease his profit by 29 086 CHF, the passengers could gain 141 897 CHF (similar values for the cyclic and the non-cyclic models). We would encourage such a trade-off – for one it secures the realized demand to be equal to the forecasted demand and for two, the TOC could charge the profit loss to the passengers, in this case each passenger would have to pay 6.5 CHF, while saving on average 32 CHF.

Please note that the minimum amount of trains needed to serve the passengers is 36, the maximum amount being 48 and the actual number of trains used being 49 (Table 1). The one unscheduled train is the last train on the line 9.

5.3 Sensitivity Analysis

In this section, we further investigate if a scenario, where the difference between the cyclic and the non-cyclic timetable would be significant, exists. We know that in terms of the maximum profit (as mentioned in the previous section) the models will give the same result. Thus we focus on the passenger cost.

The parameter of our cost function (Equation 1) that might be the most sensitive to the cyclicity is the scheduled delay. This parameter can be interpreted as a sensitivity to the time: busy people might have high values of this parameter, whereas people traveling for leisure might have lower values. We have tested 2 additional settings of this value: half of the original value and double of the original value. The results (in Table 5) show that with higher values of the scheduled delay parameters, the difference between the cost of the cyclic and the non-cyclic model is getting larger, however it is still marginal in the terms of the whole cost. With the higher scheduled delay parameter, the passengers are more willing to take the slower trains, in order to reach their destinations on time. This has been projected into the higher TOC profit – the amount of trains scheduled is similar, but the amount of rolling stock used is lower as the demand is now spread more equally to the slower trains.

Another attribute that might be influenced by the cylicity is the demand. At first, we explore the effect of the amount of the passengers on the passenger cost. As it can be seen in Figure 8, the evolution of the passenger cost is almost linear. The cyclic and the non-cyclic model give similar result in all of the instances. We did not explore instances with larger demand than as in our case study, due to the fact that the network is almost saturated (maximum amount of trains scheduled) at the ϵ of 100. Thus no further improvement is expected. As a by-product of this analysis, we can plot the evolution of the TOC profit. We have done so for the current model

	Current			Cyclic			Non-Cyclic		
SD	half	original	double	half	original	double	half	original	double
profit [CHF]	140 358	146 099	149 155	140 176	144 492	145 335	139 848	144 971	146 013
cost [CHF]	135 596	148 197	169 893	124 431	138 140	163 557	122 132	135 455	158 772
lb [CHF]	119 997	132 489	155 604	98 396	99 153	100 395	97 504	97 706	98 009
gap [%]	11.50	10.60	8.41	20.93	28.22	38. 62	20.17	27.87	38.27
gap [CHF]	15 599	15 708	14 289	26 045	38 987	63 162	24 628	37 749	60 763
time [s]	7 200	7 200	7 200	7 200	7 200	7 200	7 200	7 200	7 200
drivers [-]	48	48	47	48	48	48	48	47	47
rolling stock [-]	96	84	81	96	87	84	96	85	82

Table 5: *Sensitivity of the scheduled delay at ϵ level 100*

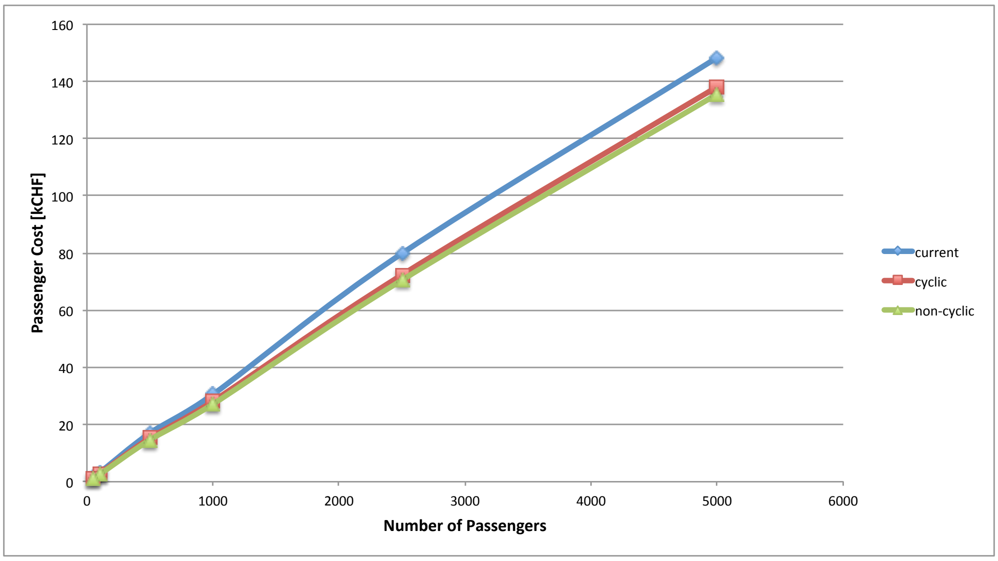

Figure 8: *The passenger cost as a function of the demand*

in Figure 9 (the profits are almost the same for the other two models). As it can be seen, this function is as well almost linear and the TOC becomes profitable at around 2 000 passengers in the network. The detailed computational results of the demand levels can be found in Tables 8, 9 and 10 in the Appendix 10.

As a last step of our analysis, we create a different demand distribution. We want to test, if a disrupted irregular demand would be better served by the non-cyclic timetable. The modification, in respect to the original hourly distribution values, can be seen in Figure 6. The results in Table 6 show that the difference between the cyclic and the non-cyclic model is in fact larger (about 6 000 CHF). However in the terms of the total cost, the savings are marginal.

Overall, we were not able to find a scenario, where the non-cyclic timetable would perform significantly better than the cyclic timetable.

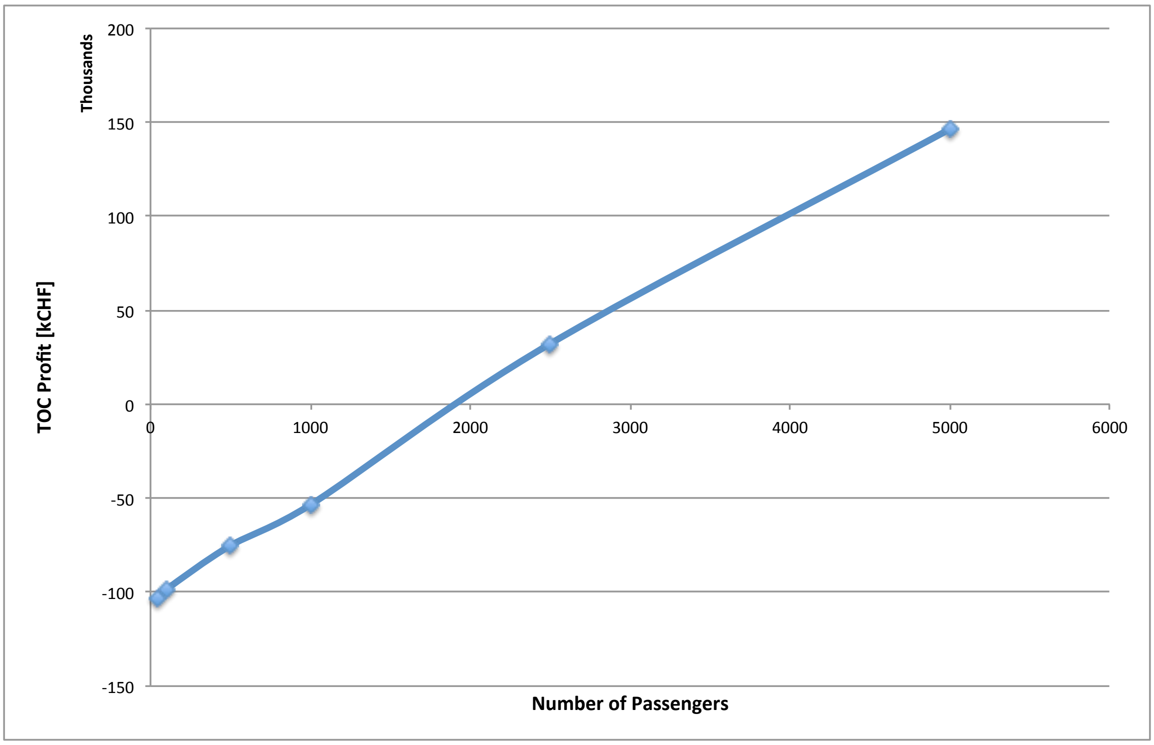

Figure 9: *The TOC profit as a function of the demand (current model)*

	Current	Cyclic	Non-cyclic
profit [CHF]	139 434	139 823	137 154
cost [CHF]	140 404	131 769	125 681
lb [CHF]	128 595	96 393	95 176
gap [%]	8.41	26.85	24.27
gap [CHF]	11 809	35 376	30 505
time [s]	7 200	7 200	7 200

Table 6: *The computational results for the irregular demand at ϵ level 100*

6 Conclusions and Future Work

In this research, we survey the literature on the current planning horizon for the railway passenger service and we identify a gap in the planning horizon – demand based (ideal) timetables. We then define a new way, how to measure the quality of a timetable from the passenger point of view and introduce a definition of such an ideal timetable. We combine this passenger based approach with the TOC point of view (profit maximization). We present a formulation of a mixed integer linear problem that can design such timetables. Since our objective function consists of two objectives, we turn the passenger cost minimization into the ϵ constraint. The new Ideal Train Timetabling Problem fits into the current planning horizon of railway passenger service and is in line with the new market structure and the current trend of putting passengers back into consideration, when planning a railway service.

The novel approach not only designs timetables that fit the best the passengers, but that also creates by itself connections between two trains, when needed. Moreover, the output consists of the routing of the passengers and thus the train occupation can be extracted and be used efficiently, when planning the rolling stock assignment (*i.e.* the Rolling Stock Planning Problem). The ITTP can create both non-cyclic and cyclic timetables.

We test the model on a semi-real data of the S-train network of Canton Vaud in Switzerland. The results show an average trade-off, between the most profitable train service and the least passenger costly service, of 30 000 CHF profit against 150 000 CHF of cost savings. Our model was able to find a better timetable compared to the current SBB timetable only in terms of the passenger cost, where the achieved savings were around 10 000 CHF. The difference between the cyclic and the non-cyclic timetable reveals itself only on the best passenger cost, where the difference is approximately 3 000 CHF benefiting the non-cyclic timetable.

Further, In our sensitivity analysis, we have focused on exploiting the difference between the cyclic and the non-cyclic timetable. Overall, we were not able to find a larger difference than around 6 000 CHF, in case of time sensitive passenger (high value of the scheduled delay parameters). These results justify the usage (or superiority) of the cyclic timetables, which are considered better for the passengers as it is easier to remember just the cycle instead of the whole timetable.

In the future work, we will focus on efficient solving of the problem and extension of the planning horizon, *i.e.* to be able to solve the problem for a whole day. This would allow us to explore, if the non-cyclic timetables could perform better off-peak hours and in the context of the whole day. The new definition of a quality of a timetable (the passenger point of view) creates a lot of opportunities for future research: efficient handling of the TOC's fleet, better delay management, robust train timetabling passenger-wise, *etc.*

7 References

Axhausen, K. W., S. Hess, A. König, G. Abay, J. J. Bates and M. Bierlaire (2008) Income and distance elasticities of values of travel time savings: New swiss results, *Transport Policy*, **15** (3) 173 – 185, ISSN 0967-070X.

Ben-Akiva, M. and S. Lerman (1985) *Discrete Choice Analysis*, The MIT Press, Cambridge Massachusetts.

Brannlund, U., P. O. Lindberg, A. Nou and J. E. Nilsson (1998) Railway timetabling using lagrangian relaxation, *Transportation Science*, **32** (4) 358–369, ISSN 1526-5447.

Burdett, R. and E. Kozan (2010) A sequencing approach for creating new train timetables, *OR Spectrum*, **32** (1) 163–193, ISSN 0171-6468.

Cacchiani, V., A. Caprara and M. Fischetti (2012) A lagrangian heuristic for robustness, with an application to train timetabling, *Transportation Science*, **46** (1) 124–133, ISSN 1526-5447.

Cacchiani, V., A. Caprara and P. Toth (2008) A column generation approach to train timetabling on a corridor, *4OR*, **6** (2) 125–142, ISSN 1619-4500.

Cacchiani, V., A. Caprara and P. Toth (2010a) Non-cyclic train timetabling and comparability graphs, *Operations Research Letters*, **38** (3) 179 – 184, ISSN 0167-6377.

Cacchiani, V., A. Caprara and P. Toth (2010b) Scheduling extra freight trains on railway networks, *Transportation Research Part B: Methodological*, **44** (2) 215 – 231, ISSN 0191-2615.

Cacchiani, V., A. Caprara and P. Toth (2013) Finding cliques of maximum weight on a generalization of permutation graphs, *Optimization Letters*, **7** (2) 289–296, ISSN 1862-4472.

Caimi, G., M. Fuchsberger, M. Laumanns and K. Schupbach (2007) Periodic railway timetabling with event flexibility.

Caprara, A. (2010) Almost 20 years of combinatorial optimization for railway planning: from lagrangian relaxation to column generation, paper presented at the *Proceedings of the 10th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems*, vol. 14 of *OpenAccess Series in Informatics (OASIcs)*, 1–12, Dagstuhl, Germany, ISBN 978-3-939897-20-0, ISSN 2190-6807.

Caprara, A., M. Fischetti and P. Toth (2002) Modeling and solving the train timetabling problem, *Operations Research*, **50** (5) 851–861.

Caprara, A., L. G. Kroon, M. Monaci, M. Peeters and P. Toth (2007) Passenger railway optimization, in C. Barnhart and G. Laporte (eds.) *Handbooks in Operations Research and Management Science*, vol. 14, chap. 3, 129–187, Elsevier.

Caprara, A., M. Monaci, P. Toth and P. L. Guida (2006) A lagrangian heuristic algorithm for a real-world train timetabling problem, *Discrete Applied Mathematics*, **154** (5) 738 – 753, ISSN 0166-218X. IV ALIO/EURO Workshop on Applied Combinatorial Optimization.

Carey, M. and D. Lockwood (1995) A model, algorithms and strategy for train pathing, *The Journal of the Operational Research Society*, **46** (8) pp. 988–1005, ISSN 01605682.

Chierici, A., R. Cordone and R. Maja (2004) The demand-dependent optimization of regular train timetables, *Electronic Notes in Discrete Mathematics*, **17** (0) 99 – 104, ISSN 1571-0653. Workshop on Graphs and Combinatorial Optimization.

de Keizer, B., K. Geurs and G. Haarsman (2012) Interchanges in timetable design of railways: A closer look at customer resistance to interchange between trains., paper presented at the *Proceedings of the European Transport Conference, Glasgow, 8-10 October 2012 (online)*, October 2012.

Erol, B. (2009) Models for the Train Timetabling Problem, Master Thesis, TU Berlin.

Fischer, F., C. Helmberg, J. Janšen and B. Krostitz (2008) Towards solving very large scale train timetabling problems by lagrangian relaxation, paper presented at the *ATMOS 2008 - 8th Workshop on Algorithmic Approaches for Transportation Modeling, Optimization, and Systems*, Dagstuhl, Germany.

Harrod, S. S. (2012) A tutorial on fundamental model structures for railway timetable optimization, *Surveys in Operations Research and Management Science*, **17** (2) 85 – 96, ISSN 1876-7354.

Higgins, A., E. Kozan and L. Ferreira (1997) Heuristic techniques for single line train scheduling, *Journal of Heuristics*, **3** (1) 43–62, ISSN 1381-1231.

Kanai, S., K. Shiina, S. Harada and N. Tomii (2011) An optimal delay management algorithm from passengers' viewpoints considering the whole railway network, *Journal of Rail Transport Planning & Management*, **1** (1) 25 – 37, ISSN 2210-9706. Robust Modelling of Capacity, Delays and Rescheduling in Regional Networks.

Kaspi, M. and T. Raviv (2013) Service-oriented line planning and timetabling for passenger trains, *Transportation Science*, **47** (3) 295–311.

Kroon, L. G., R. Dekker and M. J. C. M. Vromans (2007) Cyclic railway timetabling: a stochastic optimization approach, paper presented at the *Proceedings of the 4th international Dagstuhl, ATMOS conference on Algorithmic approaches for transportation modeling, optimization, and systems*, ATMOS'04, 41–66, Berlin, Heidelberg, ISBN 3-540-74245-X, 978-3-540-74245-6.

Kroon, L. G. and L. W. P. Peeters (2003) A variable trip time model for cyclic railway timetabling, *Transportation Science*, **37** (2) 198–212, ISSN 1526-5447.

Kroon, L. G., L. W. P. Peeters, J. C. Wagenaar and R. A. Zuidwijk (2014) Flexible connections in pesp models for cyclic passenger railway timetabling, *Transportation Science*, **48** (1) 136–154.

Liebchen, C. (2004) Symmetry for periodic railway timetables, *Electronic Notes in Theoretical Computer Science*, **92** (0) 34 – 51, ISSN 1571-0661. Proceedings of {ATMOS} Workshop 2003.

Liebchen, C. and R. Mohring (2002) A Case Study in Periodic Timetabling, *Electronic Notes in Theoretical Computer Science*, **66** (6) 18–31, ISSN 15710661.

Liebchen, C. and R. H. Mohring (2004) The modeling power of the periodic event scheduling problem: Railway timetablesÑand beyond, *Technical Report*, Preprint 020/2004, Mathematical Institute.

Lindner, T. and U. Zimmermann (2000) Train schedule optimization in public rail transport, *Technical Report*, Ph. Dissertation, Technische Universitat.

Lindner, T. and U. Zimmermann (2005) Cost optimal periodic train scheduling, *Mathematical Methods of Operations Research*, **62** (2) 281–295, ISSN 1432-2994.

Nachtigall, K. (1996) Periodic network optimization with different arc frequencies, *Discrete Applied Mathematics*, **69**, 1 – 17, ISSN 0166-218X.

Nachtigall, K. and S. Voget (1996) A genetic algorithm approach to periodic railway synchronization, *Computers & Operations Research*, **23** (5) 453 – 463, ISSN 0305-0548.

Odijk, M. A. (1996) A constraint generation algorithm for the construction of periodic railway timetables, *Transportation Research Part B: Methodological*, **30** (6) 455 – 464, ISSN 0191-2615.

Oliveira, E. and B. M. Smith (2000) A job-shop scheduling model for the single-track railway scheduling problem, *Technical Report*, University of Leeds.

Peeters, L. (2003) *Cyclic Railway Timetable Optimization*, ERIM Ph.D. series Research in Management, Erasmus Research inst. of Management (ERIM), ISBN 9789058920423.

Sato, K., K. Tamura and N. Tomii (2013) A mip-based timetable rescheduling formulation and algorithm minimizing further inconvenience to passengers, *Journal of Rail Transport Planning & Management*, **3** (3) 38 – 53, ISSN 2210-9706. Robust Rescheduling and Capacity Use.

Serafini, P. and W. Ukovich (1989) A mathematical for periodic scheduling problems, *SIAM J. Discret. Math.*, **2** (4) 550–581, ISSN 0895-4801.

Shafia, M. A., S. J. Sadjadi, A. Jamili, R. Tavakkoli-Moghaddam and M. Pourseyed-Aghaee (2012) The periodicity and robustness in a single-track train scheduling problem, *Appl. Soft Comput.*, **12** (1) 440–452, ISSN 1568-4946.

Small, K. A. (1982) The scheduling of consumer activities: Work trips, *The American Economic Review*, **72** (3) pp. 467–479, ISSN 00028282.

Vansteenwegen, P. and D. V. Oudheusden (2006) Developing railway timetables which guarantee a better service, *European Journal of Operational Research*, **173** (1) 337 – 350, ISSN 0377-2217.

Vansteenwegen, P. and D. V. Oudheusden (2007) Decreasing the passenger waiting time for an intercity rail network, *Transportation Research Part B: Methodological*, **41** (4) 478 – 492, ISSN 0191-2615.

Wardman, M. (2004) Public transport values of time, *Transport Policy*, **11** (4) 363 – 377, ISSN 0967-070X.

8 The Full ITTP Model

$$
\begin{aligned}
\max \; & \sum_{l\in L}\sum_{v\in V^l}\sum_{g\in G^l} o^l_{vg}\cdot b_g - \sum_{l\in L}\sum_{v\in V^l}\left(\alpha^l_v\cdot e\cdot k^l + u^l_v\cdot n\cdot k^l\right) \\
& \sum_{i\in I}\sum_{t\in T_i} D^t_i\cdot C^t_i \le \epsilon, \\
& \sum_{p\in P_i} x^{tp}_i = 1, && \forall i\in I, \forall t\in T_i, \\
& \sum_{v\in V^l} y^{tplv}_i = 1, && \forall i\in I, \forall t\in T_i, \forall p\in P_i, \forall l\in L^p, \\
& \left(d^l_v - d^l_{v-1}\right) = c\cdot z^l_v, && \forall l\in L, \forall v\in V^l : v>1, \\
& o^l_{vg} = \sum_{i\in I}\sum_{t\in T_i}\sum_{p\in P_i} y^{tplv}_i\cdot D^t_i, && \forall l\in L, \forall v\in V^l, \forall g\in G^l, \\
& u^l_v\cdot\beta \ge o^l_{vg}, && \forall l\in L, \forall v\in V^l, \forall g\in G^l, \\
& \alpha^l_v\cdot j \ge u^l_v, && \forall l\in L, \forall v\in V^l, \\
& s^t_i \ge s^{tp}_i - M\cdot\left(1 - x^{tp}_i\right), && \forall i\in I, \forall t\in T_i, \forall p\in P_i, \\
& s^{tp}_i \ge f_2\cdot\left(\left(d^{|L|}_v + h^{|L|}_i + r^{p|L|}_i\right) - t\right) \\
& \qquad -M\cdot\left(1 - y^{tp|L|v}_i\right), && \forall i\in I, \forall t\in T_i, \forall p\in P_i, \forall v\in V^{|L|}, \\
& s^{tp}_i \ge f_1\cdot\left(t - \left(d^{|L|}_v + h^{|L|}_i + r^{p|L|}_i\right)\right) \\
& \qquad -M\cdot\left(1 - y^{tp|L|v}_i\right), && \forall i\in I, \forall t\in T_i, \forall p\in P_i, \forall v\in V^{|L|}, \\
& w^t_i \ge w^{tp}_i - M\cdot\left(1 - x^{tp}_i\right), && \forall i\in I, \forall t\in T_i, \forall p\in P_i, \\
& w^{tp}_i = \sum_{l\in L^p\setminus 1} w^{tpl}_i, && \forall i\in I, \forall t\in T_i, \forall p\in P_i, \\
& w^{tpl}_i \ge \left(\left(d^l_v + h^{pl}_i\right) - \left(d^{l'}_{v'} + h^{pl'}_i + r^{pl'}_i + m\right)\right) && \forall i\in I, \forall t\in T_i, \forall p\in P_i, \forall l\in L^p : \\
& \qquad -M\cdot\left(1 - y^{tpl'v'}_i\right) - M\cdot\left(1 - y^{tplv}_i\right), && l>1, l'=l-1, \forall v\in V^l, \forall v'\in V^{l'}, \\
& w^{tpl}_i \le \left(\left(d^l_v + h^{pl}_i\right) - \left(d^{l'}_{v'} + h^{pl'}_i + r^{pl'}_i + m\right)\right) && \forall i\in I, \forall t\in T_i, \forall p\in P_i, \forall l\in L^p : \\
& \qquad +M\cdot\left(1 - y^{tpl'v'}_i\right) + M\cdot\left(1 - y^{tplv}_i\right), && l>1, l'=l-1, \forall v\in V^l, \forall v'\in V^{l'}, \\
& C^t_i = q_v\cdot q_w\cdot w^t_i + q_v\cdot a\cdot\sum_{p\in P} x^{tp}_i\cdot\left(|L^p|-1\right) \\
& \qquad +q_v\cdot\sum_{p\in P}\sum_{l\in L^p} r^{pl}_i\cdot x^{tp}_i + q_v\cdot s^t_i, && \forall i\in I, \forall t\in T_i, \\
& C^t_i \ge 0, && \forall i\in I, \forall t\in T_i, \\
& d^l_v \ge 0, && \forall l\in L, \forall v\in V^l, \\
& x^{tp}_i \in (0,1), && \forall i\in I, \forall t\in T_i, \forall p\in P_i,
\end{aligned}
$$

$$
\begin{aligned}
y_i^{tplv} &\in (0,1), && \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p, \forall v \in V^l, \\
o_{vg}^l &\geq 0, && \forall l \in L, \forall v \in V^l, \forall g \in G^l, \\
u_v^l &\in (0,j), && \forall l \in L, \forall v \in V^l, \\
\alpha_v^l &\in (0,1), && \forall l \in L, \forall v \in V^l, \\
z_v^l &\in \mathbb{N}, && \forall l \in L, \forall v \in V^l, \\
s_i^t &\geq 0, && \forall i \in I, \forall t \in T_i, \\
s_i^{tp} &\geq 0, && \forall i \in I, \forall t \in T_i, \forall p \in P_i, \\
w_i^t &\geq 0, && \forall i \in I, \forall t \in T_i, \\
w_i^{tp} &\geq 0, && \forall i \in I, \forall t \in T_i, \forall p \in P_i, \\
w_i^{tpl} &\geq 0, && \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p.
\end{aligned}
$$

9 Parameter Settings

M = 720 (min)
m = 4 min
c = 60 min
r_i^{pl} = www.sbb.ch
h_i^{pl} = www.sbb.ch
D_i^t = as described in Section 5
q_t = 27.81 CHF per hour
q_w = 2.5
f_1 = 0.5
f_2 = 1
a = 10 min
b_g = www.sbb.ch
e = 15 CHF per train per km
n = 15 CHF per train unit per km
β = 380 passengers
j = 2 train units
k^l = www.maps.google.ch
ϵ = various

10 Extra Results

demand [pax]	50	100	500	1 000	2 500	5 000
profit [CHF]	-103 124	-98 511	-75 183	-53 500	32 216	146 099
cost [CHF]	1 609	3 142	17 054	30 552	79 893	148 197
lb [CHF]	1 609	3 142	17 053	30 552	73 270	132 489
gap [%]	0.00	0.00	0.01	0.00	8.29	10.60
gap [CHF]	0	0	1	0	6 623	15 708
time [s]	2	2	31	69	7 200	7 200

Table 8: *Results of the current model for a different sizes of the demand*

demand [pax]	50	100	500	1 000	2 500	5 000
profit [CHF]	-103 157	-98 509	-75 159	-53 426	32 017	144 492
cost [CHF]	1 221	2 760	15 324	28 162	72 397	138 140
lb [CHF]	1 052	2 228	11 375	21 315	55 266	99 153
gap [%]	13.84	19.28	25.77	24.31	23.66	28.22
gap [CHF]	169	532	3 949	6 847	17 131	38 987
time [s]	7 200	7 200	7 200	7 200	7 200	7 200

Table 9: *Results of the cyclic model for a different sizes of the demand*

demand [pax]	50	100	500	1 000	2 500	5 000
profit [CHF]	-103 142	-98 622	-75 071	-53 498	31 153	144 971
cost [CHF]	1 078	2 514	14 669	27 167	70 564	135 455
lb [CHF]	1 018	2 159	11 196	20 999	54 387	97 706
gap [%]	5.57	14.12	23.68	22.70	22.93	27.87
gap [CHF]	60	355	3 473	6 168	16 177	37 749
time [s]	7 200	7 200	7 200	7 200	7 200	7 200

Table 10: *Results of the non-cyclic model for a different sizes of the demand*