Dynamic Traffic Assignment with Macroscopic Fundamental Diagrams

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Abstract

Dynamic traffic management strategies (e.g. perimeter control, gating) that benefit from macroscopic fundamental diagram (MFD) dynamics provided promising results regarding their effects on network capacity and performance. However, this raises the question of route choice behavior in case of heterogeneous urban networks, where different parts are modeled with separate MFD functions. In this study, we incorporate a route choice model into the MFD, and establish stochastic dynamic user equilibrium (SDUE) conditions. This study satisfies equilibrium conditions through two main components; stochastic network loading and averaging with method of successive averages (MSA). The loading procedure incorporates random sampling of origin and destination points inside the regions and a logit-based traffic assignment component to account for trip length distributions and perception errors in travel time, respectively. The results taken from stochastic loading procedure are, then, processed through MSA, which is an effective solution heuristic highly implemented in simulation-based DTA, and SDUE conditions are achieved in the network.

Keywords

DTA, MFD, user equilibrium, MSA

1 Introduction

Travelers' decisions, such as when to make a trip, which mode to use, where to go and which way to get there, are the main factors that compose traffic conditions on any link or on any intersection in an urban area and freeway network. These decisions, particularly when to make a trip (i.e. departure time) and which way to get to the destination (i.e. route choice), highly depend on location of congestion pockets and traffic conditions in the network. Congestion at any point depends, in turn, on traffic flow through that point. The need to model the interaction between congestion and travel decisions has led to the development of thinking on traffic equilibrium. Transportation engineers or urban planners have to consider traffic equilibrium phenomenon to predict the impacts of given transportation scenarios, which can describe transportation infrastructure, control strategies and travel demand, either in their existing or projected states.

Two general approaches can be found in the literature to deal with traffic equilibrium problems; static and dynamic traffic assignment models. Static models, where time-varying flows and costs are not considered, represent traffic dynamics by link performance functions and they cannot be used to evaluate dynamic traffic management and control strategies. A stable equilibrium condition is reached in static case *when no traveler can improve his travel time by unilaterally changing routes* (Wardrop, 1952). This is the definition of user-equilibrium (UE) condition. On the other hand, dynamic traffic assignment (DTA) models remove the assumption of static models, and deal with time-varying flows to adequately capture traffic dynamics. Ran *et al.* (1996) extended the equilibrium definition to the dynamic case: *For each OD pair, if the actual travel times experienced by travelers departing at the same time are equal and minimal, then the dynamic flow over the network is in a travel time based ideal dynamic user-optimal (DUO) state*. Note that DTA models can be further distinguished based on underlying travel choice component: the reactive dynamic user-optimal (RDUO) assignment, in which travelers choose the route with minimum cost based on instantaneous travel times (Papageorgiou, 1990, Kuwahara and Akamatsu, 1997, 2001), and the predictive dynamic user-optimal assignment (PDUO), in which travelers choose the route that minimizes their actual travel cost (Mahmassani and Peeta, 1993, Peeta and Mahmassani, 1995a, Ben-Akiva *et al.*, 1997a,b, Lo and Szeto, 2002). In the latter, travelers are assumed to know/anticipate future traffic conditions along the route through learning from past experience or through Advanced Traveler Information Systems (ATIS). Note that, aside from UE conditions, both static and dynamic assignments can be implemented to reach system-optimal (SO) conditions, which involves minimizing total travel cost (Ziliaskopoulos, 2000, Chow, 2009). However, this normative approach rather than descriptive is beyond the scope of this paper.

The relaxation of the presumptions considered in the deterministic utility maximization rule to reach UE conditions has been proposed by Daganzo and Sheffi (1977) as stochastic user equilibrium (SUE). SUE defines the equilibrium conditions where users can no longer improve their perceived utility. Traveler's perceived utility incorporates a random component which represents perception errors or randomness in system performance. SUE notion has also been extended to the dynamic case by De Palma *et al.* (1983): For each OD pair, if travel times perceived by travelers departing at the same time are equal and minimal, then the network is in dynamic stochastic user equilibrium state (DSUE).

Traffic performance models used in DTA to describe traffic flow propagation on time-varying networks include cell transmission models (Lo and Szeto, 2002), outflow models (Merchant and Nemhauser, 1978), deterministic queuing models (Ben-Akiva *et al.*, 1986, Han, 2003), whole link models (Ran and Boyce, 1996), nonlinear time models (Jayakrishnan *et al.*, 1995), and mesoscopic models where vehicles are modeled explicitly (Mahmassani and Peeta, 1993, Peeta and Mahmassani, 1995a, Ben-Akiva *et al.*, 1997a,b). Traffic performance models listed above implement a discrete modeling approach, where the elements of traffic network (e.g. links, intersections) are modeled separately, and demand is distributed along the network through hypothetical zone centroids. An alternative to this approach is the continuum modeling where the focus is on the general trend of travel choices at the macroscopic level. The continuum approach models a network as a continuum where travelers are free to choose their paths in two dimensional continuous space. As these models are mainly used for initial planning of large regions, traffic assignments have been mostly limited to static cases. Hoogendoorn and Bovy (2004) and Jiang *et al.* (2011) are the two example studies that attempt to establish DUO conditions via continuum modeling.

A homogeneous urban region (with small spatial link density distribution) can be modeled using macroscopic fundamental diagram (MFD), which provides a uni-modal, low-scatter, and demand-independent relationship between network vehicle density and space-mean flow (Geroliminis and Daganzo, 2008). This can be considered a continuum modeling approach rather than discrete. However, urban transportation networks exhibit uneven distribution of congestion which leads to a scattered flow-density relationship. Heterogeneity in congestion distribution can affect the shape/scatter or even the existence of MFD (Buisson and Ladier, 2009, Geroliminis and Sun, 2011). By using a grid network and considering inhomogeneity of traffic as an independent variable, Mazloumian *et al.* (2010) shows that MFD remains well-defined in sub-regions of the urban network. These results are very critical, because MFD concept can be useful for heterogeneously loaded cities, if the network can be partitioned into small homogenous regions. Ji and Geroliminis (2012) develop a partitioning mechanism to minimize the variance of link densities while maintaining a spatially compact shape. Resulting sub-regions can be used to develop macroscopic traffic control strategies; e.g. perimeter control. Geroliminis *et al.* (2013) develops perimeter control strategies for a network with two regions (two MFD's).

They implement model predictive control schemes to determine the optimal inter-transfer flows at the boundary of two regions, where traffic is controlled by signals. However, their approach ignores the fact that drivers might change their routes (i.e. sequence of regions to follow) as a result of change in traffic signals, i.e. route choice.

In this paper, we incorporate a route choice model into the MFD, and establish DUO conditions. These conditions are satisfied within a DTA framework, and their effects on network capacity and performance are tested. MFD approach requires aggregate modeling of traffic flow within urban regions, and it describes traffic flow propagation within or between the regions using average trip lengths. However, average trip length is not informative enough to build a route choice framework; instead, trip length distributions (TLD) within each region and for each OD (i.e. origin and destination regions) must be considered to ensure DUO conditions in the system. This study does not explicitly calculate TLD's, but it deals with them in an iterative way within stochastic network loading procedure. This loading procedure incorporates also a logit-based traffic assignment component to account for perception errors in travel time. The results taken from this iterative stochastic loading procedure are, then, processed through an averaging procedure called method of successive averages (MSA). MSA is an effective solution heuristic which is highly implemented in simulation-based DTA (Peeta and Mahmassani, 1995b). Given that it does not require derivative information for the flow-cost mapping function, MSA can be easily adapted to simulation studies. MSA uses the simulator (i.e. MFD dynamics in this case) in each iteration to project future traffic information as part of the direction finding mechanism in searching for a solution. As simulators take significant amount of time, simulation-based DTA strategies are not suitable for real-time deployment (Peeta and Mahmassani, 1995a). On the other hand, MFD approach, even in case of large-network modeling, can get along with a small graph network with few nodes and links. Therefore, this study does not suffer from feasibility issues in real-time deployment. In addition, the approach presented in this paper can be considered to include two stochastic components; TDL's within the urban regions, and logit-based traffic assignment in loading procedure to represent perception errors and randomness in the system. In that respect, this approach can be regarded as DSUE rather than its deterministic ancestor DUE.

2 Traffic Modeling of a Multi-Region Urban Network

The output of MFD in homogenous urban region r, $P_r(\eta_r^\tau)$ [veh.km/h] represents the production corresponding to the accumulation η_r^τ [veh] at time τ. $V_r^\tau = (P_r(\eta_r^\tau))/(\eta_r^\tau)$ [km/h] represents the speed in region r, and trip completion rate is calculated to be $O_r^\tau = (P_r(\eta_r^\tau))/(l_r^\tau)$ [veh/h], considering the average trip length l_r^τ at time τ.

Consider a city partitioned in N regions. Denote by r=1,2,...,N a region in the network, η_r^τ its total accumulation at time τ, and $\eta_{o,d}^{p,r,\tau}$ the number of vehicles in region r at time τ with origin o, destination d, and path p (sequence of regions). Note that there are multiple paths for an OD pair, travelers have the chance to switch to alternative routes, and number of paths is specific to the OD pair.

Trip completion rate $O_{o,d}^{p,r,\tau}$ for the vehicles in region r at time τ with origin o, destination d, and path p is calculated by $O_{o,d}^{p,r,\tau} = \eta_{o,d}^{p,r,\tau}/\eta_r^\tau.O_r^\tau$ or $O_{o,d}^{p,r,\tau} = V_r^\tau.(\eta_{o,d}^{p,r,\tau})/(l_{o,d}^{p,r,\tau})$. Note that $l_{o,d}^{p,r,\tau}$ represents the average trip length crossed in region r by the people who travel between origin o and destination d, using path p at time τ. The dynamic model used in this study removes the assumption of constant trip length with time, and accounts for the change in trip length with respect to route choice decisions in the network. For instance, considering traffic conditions in the network, travellers who start their trips from different parts of the region can take different decisions causing a significant change in the average trip length.

Denote $q_{o,d}^\tau$ the exogenous demand generated at time τ, for origin o and destination d. As there are multiple paths connecting the same OD pair, demand will be distributed among them in a way that satisfies DUE conditions. The sum of path-specific flows $q_{o,d}^{p,\tau}$, computed through DTA, should be equal to $q_{o,d}^\tau$. Let $q_{r\to p^+(r)}^{o,d,\tau}$ be transferring flow from region r to region $p^+(r)$ which is the next reservoir in the sequence described by path p, $x^*_{r,p^+(r)}$ be a control variable for inflow capacity from r to $p^+(r)$, which can be set using perimeter control strategies. The maximum inflow or inflow capacity to region r from the previous region $p^-(r)$ in path p, is a function of accumulation η_r^τ and it is described by the entrance function $C_{p^-(r),r}(\eta_r^\tau)$. Entrance function describes the fact that accumulation in region r can restrict the inflow along the periphery. In addition, we assume that path decision is taken at the beginning of the trip and it is fixed throughout the trip. However, one can also update path decision at every node (i.e. region). Dynamic equations are listed below. Note that time τ is omitted from the equations.

$$\frac{d\eta_{o,d}^{p,r}}{d\eta} = f(o,r).q_{o,d}^{p} - q_{r\to p^+(r)}^{o,d} + q_{p^-(r)\to r}^{o,d} \tag{1}$$

where

$$f(o,r) = \begin{cases} 1 & : o = r \\ 0 & : \text{otherwise} \end{cases} \tag{2}$$

$$q_{r\to p^+(r)}^{o,d} = min(x^*_{r,p^+(r)}.a_{r\to p^+(r)}^{o,d}, C_{r,p^+(r)}.a_{r\to p^+(r)}^{o,d}, O_{o,d}^{p,r}) \tag{3}$$

Eq. 1 simply defines that the rate $\eta_{o,d}^{p,r}$ equals to the inflow from the previous reservoir in path p, minus the outflow to the next reservoir in path p, and plus the generated OD demand if region r is the same as origin reservoir o. Note that the entrance function $C_{r,p^+(r)}$ is assumed independent of the accumulation η_r. The reason is that entrance function is invariant for the values of accumulation below the critical value. However, any control strategy will try to avoid accumulations in the undesirable congested regime. Therefore, $x^*_{r,p^+(r)} < C_{r,p^+(r)}$ in the congested regime, and $O_r < C_{r,p^+(r)}$ in the uncongested regime. $a^{o,d}_{r\rightarrow p^+(r)}$ in Eq. 3 is the fraction of inflow capacity $C_{r,p^+(r)}$ or control rate $x^*_{r,p^+(r)}$ from region r to $p^+(r)$, for vehicles with next region $p^+(r)$, and can be computed with the following formula;

$$a^{o,d}_{r\rightarrow p^+(r)} = \frac{\eta^{p,r}_{o,d}/l^{p,r}_{o,d}}{\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{N^p_{od}} g(p,r,t).\eta^{t,r}_{i,j}/l^{t,r}_{i,j}} \tag{4}$$

where

$$g(p,r,t) = \begin{cases} 1 & : p^+(r) = t^+(r) \\ 0 & : \text{otherwise} \end{cases} \tag{5}$$

N^p_{od} is the number of paths between origin o and destination d, and $g(p,r,t)$ is a binary variable with value equal to 1 if the next reservoir in the paths p and t is the same.

3 Stochastic Network Loading

Stochastic network loading concept has been developed for the approach in which travellers' perception of travel time is assumed to be random. These models are special applications of discrete choice models, and they are mainly based on logit or probit models. Logit-based traffic assignment can be implemented without enumeration of paths using STOCH algorithm in the static case (Dial, 1971) or DYNASTOCH algorithm in the dynamic case (Ran and Boyce, 1996). On the other hand, probit based assignment, which accounts for correlation between path travel times, is based on Monte Carlo simulation of perceived link travel times. The approach presented in this study, initially, does not assign any explicit travel time distribution on the paths. Instead, by assuming origins and destinations are distributed in the continuous space defined by the region, the model generates starting and ending points for trips within the same region or in different regions. The algorithm, then, computes lowest travel times on alternative paths for the generated start and end points (this part will be further discussed later in the paper), and takes a path decision based on logit discrete choice model, where, finally, an independent Gumbel distribution is assumed for each path travel time.

The loading algorithm can be summarized as follows:

Step 0: *Initialization.* Set n=1.

Step 1: *Sampling and travel time computation.*

- For each (o, d, τ) triplet, sample origin and destination points within the first and last regions of the trip.
- For each path p between origin o and destination d, identify time-dependent shortest path at departure time τ. As path p represents only the sequence of regions, there are many possible link sequences that comply with this definition. In this step, we choose the sequence of links that leads to the lowest experienced travel time, $T_{o,d}^{p,\tau(n)}$, among all possible link sequences that suit region sequence, p. Also, keep $l_{o,d}^{p,r,\tau(n)}$ in the memory to calculate average trip length later.

Step 2: *Logit assignment.* Based on lowest travel times $T_{o,d}^{p,\tau(n)}$ calculated in the previous step, compute probability of using path p with the following logit formula; $P_{o,d}^{p,\tau(n)} = e^{T_{o,d}^{p,\tau(n)}} / \sum_k e^{T_{o,d}^{k,\tau(n)}}$.

Step 3: *Flow averaging.* Let $q_{o,d}^{(p,\tau(n)} = \left((n-1) * q_{o,d}^{p,\tau(n-1)} + P_{o,d}^{p,\tau(n)} * q_{o,d}^{\tau}\right) / n$ for each (o, d, p, τ) quartet.

Step 4: *Stopping test.*

- Evaluate the following for each (o, d, p, τ) quartet $\left[q_{o,d}^{p,\tau(n)} - q_{o,d}^{p,\tau(n-1)}\right] \geq \epsilon$.
- Calculate $N(\epsilon)$ the number of cases for which the above criterion is violated.
- If $N(\epsilon) \geq \psi$, set $n = n + 1$ and go to Step 1.
- Otherwise, finish the procedure, calculate average trip length for each (o, d, p, τ, r) with the following formula: $l_{o,d}^{p,r,\tau(*)} = \left(\sum_{k=1}^{n} l_{o,d}^{p,r,\tau(k)} \cdot P_{o,d}^{p,\tau(k)}\right) / \sum_{k=1}^{n} P_{o,d}^{p,\tau(k)}$, and assign $q_{o,d}^{p,\tau(*)} = q_{o,d}^{p,\tau(n)}$ for each (o, d, p, τ) quartet.

Note that the loading procedure incorporates two stochastic components; random sampling of origin and destination points, and logit assignment. The former is implemented to account for trip length distributions within and between regions. The original MFD approach assumes constant average trip lengths to achieve traffic propagation and to determine outflow values. This assumption has been revised in this study with the consideration of route choice behaviour. The latter stochastic component has been incorporated in the algorithm to address travellers' perception of travel time. Since MFD requires aggregate traffic modelling in the spatial aspect, travellers may perceive traffic conditions differently depending on their location in the region and can take different route choice decisions based on these perceptions. Logit assignment is expected to overcome the problems that arise from this phenomenon.

4 Method of Successive Averages (MSA)

The MSA algorithm has been used in both static and dynamic network equilibrium problems as an incremental assignment type heuristic (Daganzo and Sheffi, 1977, Mahmassani and Peeta, 1993). The method is based on predetermined step sizes along the descent direction. In other words, step size is not determined with respect to the characteristics of the current solution, which requires derivative information. Instead, it is determined a priori. Therefore, the MSA stands as one of the most effective solution heuristics in case the derivative information is difficult to be acquired. Powell and Sheffi (1982) provided a formal proof of convergence for the MSA relying on general convergence proof by Blum (1954). Nevertheless, traffic assignment literature on MSA relies on small 'toy' networks, because it is known to suffer from slow convergence problems in case of large-scale real transportation networks. On the other hand, this study, using the MFD approach in the modeling of urban networks, is not expected to suffer from these limitations.

The MSA algorithm can be summarized as follows:

Step 0: *Initialization.* Perform a stochastic network loading based on initial region speeds $\{V_r^{\tau(0)}\}$. This generates a set of path flows $\{q_{o,d}^{p,\tau(1)}\}$. Set m=1.

Step 1: *Update.* Implement MFD dynamics to set $\{V_r^{\tau(m)}\} = f\left(\{q_{o,d}^{p,\tau(m)}\}\right)$.

Step 2: *Direction finding.* Perform a stochastic network loading based on current region speeds $\{V_r^{\tau(m)}\}$. This generates an auxiliary link flow $\{q_{o,d}^{p,\tau(*)}\}$ and trip length set $\{l_{o,d}^{p,r,\tau(*)}\}$.

Step 3: *Find new flow and trip length set.*
- For each (o, d, p, τ) quartet, set $q_{o,d}^{p,\tau(m+1)} = q_{o,d}^{p,\tau(m)} + (1/m)\left(q_{o,d}^{p,\tau(*)} - q_{o,d}^{p,\tau(m)}\right)$.
- For each (o, d, p, τ, r) quartet, set $l_{o,d}^{p,r,\tau(m+1)} = l_{o,d}^{p,r,\tau(m)} + (1/m)\left(l_{o,d}^{p,r,\tau(*)} - l_{o,d}^{p,r,\tau(m)}\right)$.

Step 4: *Stopping test.*
- Evaluate the following for each (o, d, p, τ) quartet $\left[q_{o,d}^{p,\tau(m+1)} - q_{o,d}^{p,\tau(m)}\right] \geq \epsilon$.
- Calculate $M(\epsilon)$ the number of cases for which the above criterion is violated.
- If $M(\epsilon) \geq \phi$, set $m = m + 1$ and go to Step 1. Otherwise, terminate the procedure.

The initialization step can be based on free flow speeds. $\{V_r^{\tau(0)}\}$ is the set of region speeds, as if the regions are completely empty.

Note that the convergence of MSA is not monotonic. This is because of random search direction (auxiliary values produced by stochastic network loading may sometimes point in a direction where objective function increases) and the fixed move size (predetermined step size, $\alpha_m = 1/m$, may overshoot the reduction in the objective function, as it incorporates no information related to the optimal solution neighbourhood). In addition, one can claim that convergence criterion used in MSA is forced to converge due to the nature of step size sequence $\{\alpha_m\}$. However, practical experience indicates reasonable convergence speed and existence of stable solution, before it is forced by the sequence of step size.

5 Case Study

Multi-region dynamics described in the previous section and DTA methodology to be developed within this study are first tested on a simple-network presented in Fig. 1. The network in Fig. 1 presents a case with a single destination (red dot), which is modelled using MFD dynamics. The system is partitioned into three sub-networks with different MFD functions. In this simple network, travellers have alternative paths to reach their destination. For example, people who travel between region 1 to the destination point have two alternatives; staying in region 1 (denoted as 1-D) or passing through region 3 (1-3-D). Note that, while origin points are randomly sampled within regions 1, 2 and 3, all travellers have the same destination point D.

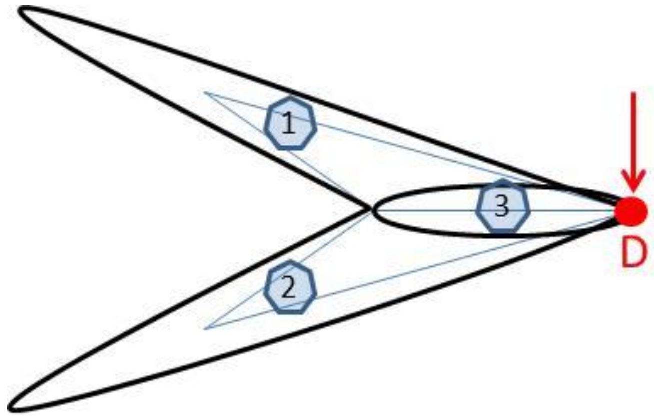

Figure 1: Single-Destination Multi-Region Network

Geroliminis and Daganzo (2008) shows that MFD can be represented by a non-symmetric unimodal curve skewed to the right (critical density is approximately 1/3 of jam density). Therefore, this study assigns a 3rd order MFD function to each region; $P_i(\eta_i) = a_i.\eta_i^3 + b_i.\eta_i^2 + c_i.\eta_i$, where a_i, b_i and c_i can be derived from empirical observations. Note that region 3 in this case study can be considered as commercial business district (CBD) of greater urban network.

Fig. 2a and b display average travel times on the alternative paths for OD pairs (1D) and (2D), respectively. Note that average travel times are calculated using the time-dependent speed measurements in the regions shown in Fig. 2c and average trip lengths depicted in Fig. 2e and f. Also, Fig. 2d provides route choice parameters depicting the proportion of OD demand using the paths 1→D and 2→D.

As region 3 is congested at the beginning of simulation, its speed is very low. Therefore, about 90% of (1D) and (2D) demand prefer to follow 1→D and 2→D, respectively, instead of going through region 3. As the time passes, the accumulation in region 1 and 2 increases, hence, travel times on alternative paths become comparable and more travelers start to choose sequence 1→3→D. θ_{1D} reaches ≃0.4 at approximately 1 h after the simulation start, which indicates that

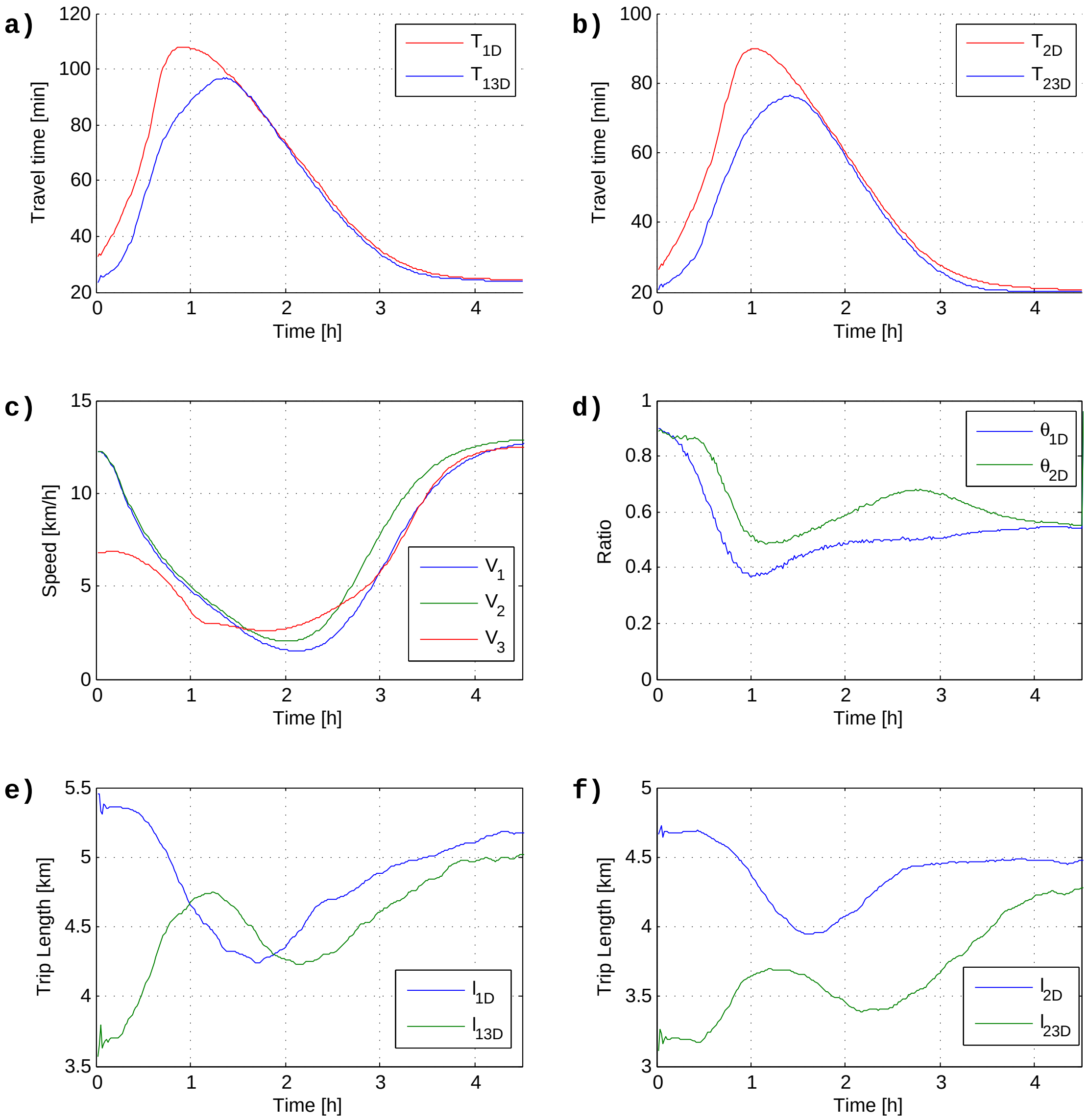

Figure 2: a. Travel times on alternative paths for OD pair (1D), b. for OD pair (2D), c. Speed measurements in the regions, d. evolution of route choice parameters, e. evolution of average trip lengths on alternative paths for OD pair (1D), f. for OD pair (2D)

about 60% of OD demand prefer to follow 1→3→D. At the end of the simulation, where speed measurements indicate almost free flow traffic conditions, both route choice parameters converge approximately to 0.5.

Note that travel times depicted in Fig. 2a and b are not conventional trip times used in discrete modeling approach. They represent average travel time experienced by the proportion of OD demand which choose the corresponding path. For instance, even though travel time on path 1→D is higher than the one on path 1→3→D at the beginning of the simulation, greater proportion of OD demand (≃0.9) choose the path 1→D. As Fig. 2e indicates, average trip length in the path 1→3→D is significant lower than the one on the alternative path at the beginning of simulation, which implies that only travelers who depart from locations close to the destination point prefer to use this path. Thus, average travel time is lower on the path 1→3→D, while only ≃10% of OD demand prefers it. As time passes, travelers who depart from other locations in the region start to choose 1→3→D, which increases the average trip length on this path. In addition, the approach presented in this study, as it aims for SDUE conditions rather than DUE conditions, is not expected to produce equal travel times on alternative paths. Instead, it presents equilibrium conditions in which people may perceive travel times differently and make route choice decisions based on their location inside the region.

6 Conclusion

Dynamic traffic management strategies (e.g. perimeter control, gating) that benefit from MFD dynamics provided promising results regarding their effects on network capacity and performance. However, this raises the question of route choice behavior in case of heterogeneous urban networks, where different parts are modeled with separate MFD functions. In this study, we incorporate a route choice model into the MFD, and establish SDUE conditions. The approach presented in this paper has two main components; stochastic network loading procedure and averaging through MSA to satisfy equilibrium conditions. The first part addresses the randomness in the network due to trip length distributions within and between regions and perception errors which may become significant in case of aggregate modeling considered in MFD approach. The second part averages auxiliary directions taken from stochastic loading procedure and establishes equilibrium conditions through MSA. Although MSA applications suffer from real-time feasibility issues in case of discrete modeling, this approach has benefited from aggregate traffic modeling brought by MFD, and has exhibited reasonable convergence speeds. Extension of this study can include a route guidance strategy, where advantages of MFD modeling and conclusions from this study can be utilized in conjunction.

7 References

Ben-Akiva, M., M. Bierlaire, J. Bottom, H. Koutsopoulos, R. Mishalani *et al.* (1997a) Development of a route guidance generation system for real-time application.

Ben-Akiva, M., A. De Palma and P. Kanaroglou (1986) Dynamic model of peak period traffic congestion with elastic arrival rates, *Transportation Science*, **20** (3) 164–181.

Ben-Akiva, M. E., H. N. Koutsopoulos, R. G. Mishalani and Q. Yang (1997b) Simulation laboratory for evaluating dynamic traffic management systems, *Journal of Transportation Engineering*, **123** (4) 283–289.

Blum, J. (1954) Multidimensional stochastic approximation methods, *Annals of Mathematical Statistics*, **25** (4) 737–744.

Buisson, C. and C. Ladier (2009) Exploring the impact of homogeneity of traffic measurements on the existence of macroscopic fundamental diagrams, *Transportation Research Record: Journal of the Transportation Research Board*, **2124** (-1) 127–136.

Chow, A. H. (2009) Dynamic system optimal traffic assignment–a state-dependent control theoretic approach, *Transportmetrica*, **5** (2) 85–106.

Daganzo, C. F. and Y. Sheffi (1977) On stochastic models of traffic assignment, *Transportation Science*, **11** (3) 253–274.

De Palma, A., M. Ben-Akiva, C. Lefevre and N. Litinas (1983) Stochastic equilibrium model of peak period traffic congestion, *Transportation Science*, **17** (4) 430–453.

Dial, R. B. (1971) A probabilistic multipath traffic assignment model which obviates path enumeration, *Transportation Research*, **5** (2) 83–111.

Geroliminis, N. and C. F. Daganzo (2008) Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings, *Transportation Research Part B: Methodological*, **42** (9) 759–770.

Geroliminis, N., J. Haddad and M. Ramezani (2013) Optimal perimeter control for two urban regions with macroscopic fundamental diagrams: A model predictive approach.

Geroliminis, N. and J. Sun (2011) Properties of a well-defined macroscopic fundamental diagram for urban traffic, *Transportation Research Part B: Methodological*, **45** (3) 605–617.

Han, S. (2003) Dynamic traffic modelling and dynamic stochastic user equilibrium assignment for general road networks, *Transportation Research Part B: Methodological*, **37** (3) 225–249.

Hoogendoorn, S. P. and P. H. Bovy (2004) Dynamic user-optimal assignment in continuous time and space, *Transportation Research Part B: Methodological*, **38** (7) 571–592.

Jayakrishnan, R., W. K. Tsai and A. Chen (1995) A dynamic traffic assignment model with traffic-flow relationships, *Transportation Research Part C: Emerging Technologies*, **3** (1) 51–72.

Ji, Y. and N. Geroliminis (2012) On the spatial partitioning of urban transportation networks, *Transportation Research Part B: Methodological*, **46** (10) 1639–1656.

Jiang, Y., S. Wong, H. Ho, P. Zhang, R. Liu and A. Sumalee (2011) A dynamic traffic assignment model for a continuum transportation system, *Transportation Research Part B: Methodological*, **45** (2) 343–363.

Kuwahara, M. and T. Akamatsu (1997) Decomposition of the reactive dynamic assignments with queues for a many-to-many origin-destination pattern, *Transportation Research Part B: Methodological*, **31** (1) 1 – 10, ISSN 0191-2615.

Kuwahara, M. and T. Akamatsu (2001) Dynamic user optimal assignment with physical queues for a many-to-many od pattern, *Transportation Research Part B: Methodological*, **35** (5) 461 – 479, ISSN 0191-2615.

Lo, H. K. and W. Szeto (2002) A cell-based variational inequality formulation of the dynamic user optimal assignment problem, *Transportation Research Part B: Methodological*, **36** (5) 421–443.

Mahmassani, H. S. and S. Peeta (1993) *Network performance under system optimal and user equilibrium dynamic assignments: Implications for advanced traveler information systems*, no. 1408.

Mazloumian, A., N. Geroliminis, D. Helbing, A. Mazloumian, N. Geroliminis and D. Helbing (2010) The spatial variability of vehicle densities as determinant of urban network capacity, *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, **368** (1928) 4627–4647.

Merchant, D. K. and G. L. Nemhauser (1978) A model and an algorithm for the dynamic traffic assignment problems, *Transportation Science*, 183–199.

Papageorgiou, M. (1990) Dynamic modeling, assignment, and route guidance in traffic networks, *Transportation Research Part B: Methodological*, **24** (6) 471 – 495, ISSN 0191-2615.

Peeta, S. and H. S. Mahmassani (1995a) System optimal and user equilibrium time-dependent traffic assignment in congested networks, *Annals of Operations Research*, **60**, 81–113.

Peeta, S. and H. S. Mahmassani (1995b) System optimal and user equilibrium time-dependent traffic assignment in congested networks, *Annals of Operations Research*, **60** (1) 81–113.

Powell, W. B. and Y. Sheffi (1982) The convergence of equilibrium algorithms with predetermined step sizes, *Transportation Science*, **16** (1) 45–55.

Ran, B. and D. Boyce (1996) *Modeling dynamic transportation networks: an intelligent transportation system oriented approach*, Springer, ISBN 9783540583608.

Ran, B., R. W. Hall and D. E. Boyce (1996) A link-based variational inequality model for dynamic departure time/route choice, *Transportation Research Part B: Methodological*, **30** (1) 31 – 46, ISSN 0191-2615.

Wardrop, J. G. (1952) Some theoretical aspects of road traffic research, *Proceedings of the Institution of Civil Engineers*, **1** (3) 325–362.

Ziliaskopoulos, A. K. (2000) A linear programming model for the single destination system optimum dynamic traffic assignment problem, *Transportation science*, **34** (1) 37–49.