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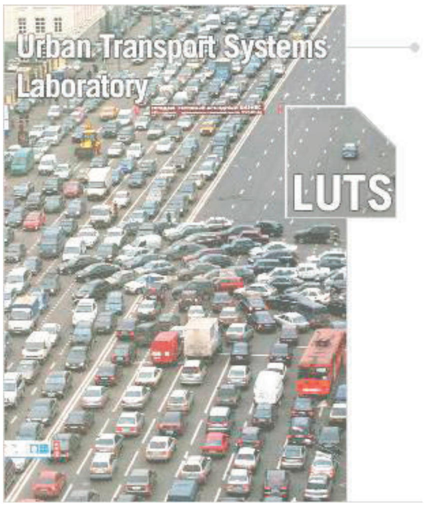

An optimization framework for the development of efficient one-way car-sharing systems

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Urban Transport Systems Laboratory (LUTS) **April 2013**

STRC | **13th Swiss Transport Research Conference**
Monte Verità / Ascona, April 24 – 26, 2013

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April 2013

Abstract

Environmental, energy, and societal considerations have given rise to the concept of shared-vehicle mobility systems. This concept postulates that the use of a fleet of vehicles made available on demand to the general public on a rental basis, can increase the mobility of certain population categories. In addition to mobility enhancement, shared-vehicle mobility systems have the potential to contribute to the sustainability of the transportation system through the decrease of environmental impacts, energy and space requirements (Duncan, 2011).

As a consequence of the promises that shared-vehicle mobility systems hold, numerous such systems have been introduced in many cities around the world (Barth *et al.,* 2006). However, most of the real-world applications of the car-sharing systems works in two ways, i.e. the vehicle should be returned to where it is rented from. Although there are some examples of one-way car-sharing systems in practice, they are not preferred by the operators because of their operational difficulties (e.g. relocations of vehicles).

In this research we aim to propose a generic model for supporting the strategic (station location and size) and tactical (fleet size) decisions of a general one-way car-sharing system, with a direct application in a case study in Nice, France. For this purpose, a mathematical model is formulated and sensitivity analysis is conducted for different parameters. As a future work, we plan to work on the operational problem which considers requests on-line and updates vehicle rosters accordingly.

Keywords

one-way car-sharing, location models, integer programming

1 Introduction

Car-sharing (also known as shared-use vehicle) systems have attracted considerable attention throughout the world (Barth *et al.,* 2006) due to their potential to improve mobility and sustainability (Duncan, 2011). These systems provide benefits both to their users and the society as a whole. Reduced personal transportation cost and mobility enhancement have been cited as the two most notable benefits to individual users. Societal benefits include reduction of parking space requirements, reduction of capacity expansion, congestion reduction, provision of affordable mobility to economically disadvantaged groups (Shaheen *et al.,* 2004, Fan *et al.,* 2008). In cases of electric shared vehicles (many examples in European cities), it can also provide significant reductions in energy and emissions.

The attractiveness of car-sharing systems is determined by the level of service offered and the cost associated with the use of the system. The level of service is influenced by the accessibility of vehicle stations, i.e. (i) how far a potential user of the system has to walk to reach a pick-up and/or a drop-off vehicle station, and (ii) the availability of vehicles at the station. The higher the accessibility of stations and availability of vehicles, the higher the level of service offered to the potential users, and hence the higher the attractiveness and potential utilization of the system. On the other hand, the station number and size, as well as the fleet size and availability of vehicles, the “right time” at the “right station”, influence the cost of establishing and operating a car-sharing system. The shared-used car system can be classified into the flexible “one-way” and the more restricted “two-way” type according to where the users can return a vehicle in a different or the same location they picked it up. The problem of ensuring vehicle availability becomes more prominent when vehicles can be used on a one-way basis, i.e. when a vehicle picked-up at a station is not necessary to be returned back to the same station. The one-way operation of the vehicles coupled with the imbalance of demand for cars, both at the origin of the trip (pick-up station) and at the destination (drop-off station), may result to a situation where vehicles are accumulated to stations that are not needed, while at the same time there is vehicle shortage at stations where more vehicles are needed. Vehicle relocation, i.e. transfer of vehicles from stations with high vehicle accumulation to stations where shortage is experienced, is a technique that has been proposed to improve the performance of one-way car-sharing systems (Barth *et al.,* 2006, Kek *et al.,* 2006, Cucu *et al.,* 2009, Fassi *et al.,* 2012). Lack of efficient vehicle relocation coupled with the need to guarantee a given level of vehicle availability, may lead to an unnecessary increase of fleet size and vehicle underutilization.

The efficient and cost-effective strategic planning and operation of station based car-sharing systems require the use of models that will determine the number and location of the service stations, the fleet size, and the dynamic allocation of vehicles to stations. These models should assist decision makers to strike an optimum balance between the level of service offered and the total cost (including vehicle relocation costs) for implementing and operating the car-sharing system.

However, the literature currently lacks a model that can consider simultaneously decisions related to the determination of station location, size and number, and fleet size, while taking into account the dynamics of vehicle relocation and balancing. Existing models either look at the location of stations without due consideration to vehicle relocation decisions (Lin and Yang, 2011), or consider the location of vehicle stations assuming that only the demand of open stations needs to be serviced (de Almeida Correia and Antunes, 2012). In the case where vehicle relocation is modeled (de Almeida Correia and Antunes, 2012), the relocation of vehicles and the associated costs are considered only at the end of the operating period, and therefore they are not taken into account in determining the fleet size.

The objective of this paper is twofold: (i) to develop and solve a mathematical model for determining the optimum fleet size and the number and location of the required stations of a car-sharing system by taking into account the dynamic repositioning of vehicles and (ii) to apply the proposed model for planning and operating a station based shared-use electric vehicle system in the city of Nice, France.

The remainder of this paper is organized as follows. Section two provides an overview of previous related work and further elaborates on the arguments justifying the need for the proposed model, section three presents the formulation and the solution approach of the proposed model, section four describes the application of the proposed model for planning and operating a station based shared-use electric vehicle system in Nice, France and section five discusses the research conclusions and provides recommendations for future research.

2 Previous Related Research

Strategic planning decisions seek to determine the number, size and location of stations, and the number of the vehicles that should be assigned to each station, in order to optimize a measure or a combination of measures of system performance. Station location models have been developed to locate bicycle stations (Lin and Yang, 2011) and car stations (de Almeida Correia and Antunes, 2012). Although the focus of our work is on shared-use station based electric car systems, in our literature review we also include a review of a model that addresses the station location of shared-use bicycles.

The problem of locating stations for shared-use bicycles has been studied recently. Lin and Yang (2011) present a model for determining the number and location of bicycle stations and the structure of the network of bicycle paths that should be developed to connect the bicycle stations. This model does not consider the daily variation of demand, and the problems arising from the dynamic accumulation/shortage of bicycles due to the variation of demand in time and space.

de Almeida Correia and Antunes (2012) addressed the optimization of car depot locations and the definition of the number of parking spaces (size of the depot) for each depot. Vehicle relocation (and the associated relocation cost) is considered only at the end of the entire operating period (i.e. day) and does not rebalance the vehicles at the end of each operating sub-interval (e.g. hour). The objective function of the model seeks to maximize the profit of the operating agency and does not consider the access and egress cost of the potential users to/from the candidate station locations.

Fan *et al.* (2008) modeled the dynamic allocation of vehicles at the end of the day among the stations of a shared-use car system to maximize profit. The fleet size, the location of the stations, and the demand for trips for a given planning horizon are known in advance. A multistage stochastic linear-model with recourse has been proposed to address this problem. A stochastic optimization method based on Monte Carlo simulation was used to solve the proposed model.

The problem of determining the fleet size and the distribution of vehicles among the stations of a car-sharing system was studied in relation to the Personal Intelligent City Accessible Vehicles (PICAVs). This system uses a homogeneous fleet of eco-friendly vehicles and allows one-way trips (Cepolina and Farina, 2012). The stations are parking lots that offer vehicle recharging services and are located at inter-modal transfer points and near major attraction sites within the pedestrian area. The number, the location, and the capacity of the stations are not determined by the model. To cope with the imbalanced accumulation of the one-way system, this model introduces the concept of supervisor. The task of the supervisor is to direct users that are flexible in returning the car to alternative stations, as to achieve a balanced operation and fulfill a maximum waiting time constraint.

Fassi *et al.* (2012) introduced a model for evaluating the performance of a network of car-sharing stations. This problem arises when the demand for car-sharing services changes (increases) and as a consequence the network of stations should be adapted to serve better the emerging demand profile. In response to this need a decision support tool was developed which allows decision makers to simulate alternative strategies. Such strategies include opening and/or closing stations, and increasing the capacity of stations. This tool is based on discrete event simulation and seeks to maximize the satisfaction level of the users and to minimize the number of the cars used.

A major decision associated with the operation of multiple station car-sharing systems is how to relocate vehicles. The vehicle relocation problem arises from the imbalanced accumulation of vehicles to stations when the car-sharing system allows their one-way use. Different strategies and models have been proposed in the literature to cope with the vehicle relocation problem.

The relocation of shared vehicles can be realized by using operating staff or it can be user based. Barth *et al.* (2004) proposed two user-based relocation strategies namely, trip-joining and trip-splitting. The trip-joining strategy is used when two users have common pick-up and drop-off stations and there is a shortage of vehicles at the pick-up station. In this case, the users are asked to share the ride. The trip-splitting strategy is used when there is a surplus of vehicles at the pick-up station and there are users that are traveling as a group. Under this condition the users are asked to use separate vehicles. The strategies were analyzed using data collected from a car-sharing system operated and through simulation. The results of the simulation model suggest that the need for vehicle relocations can be decreased by 42% by using these strategies. User based relocation can be partially achieved by introducing different pricing policies for movements that create high system imbalances (Kek *et al.,* 2009).

Kek *et al.* (2006) introduced shortest time, and inventory balancing strategies for staff based vehicle relocation. The shortest time strategy relocates cars from other stations in such a way as to minimize the travel time needed for a staff member from his/her current location to the station where the car is available plus the travel time needed from the station that the car is available to the station where the car is needed. The inventory balancing strategy relocates cars from stations with over-accumulated vehicles to stations that experience vehicle shortages. Both strategies were tested through a simulation model which was validated using data from an operational car-sharing system.

The literature review revealed that existing modeling efforts make a sharp separation between strategic and operational decisions. This means that strategic decision-making models do not integrate in their structure aspects of operational decisions, e.g. vehicle relocation, that have a significant bearing on the cost and performance of the resulting car-sharing system. On the other hand, operational models are focused on the detailed modeling of different types of relocation strategies, assuming that the location, number, and station and fleet size are given or exogenously defined.

In reality strategic, tactical, and operational decisions are interweaved and therefore there is a strong interaction between the three decision making levels. Strategic decisions are primarily related to the definition of the location, number, and size of stations and interact with the tactical decision of fleet size determination. In turn the fleet size is affected by vehicle relocation which is an operational decision. Here it is important to stress the fact that both fleet size and vehicle relocation influence the strategic level decisions. The above discussion suggests that there is a need for a model that will be able to address the strategic and tactical decisions by taking into account (at a macroscopic level) the impact of vehicle relocation.

3 Model Description

The proposed model is motivated from the planning of a station based shared-use electric vehicle system. Shared-use electric cars are used to serve trips within a given geographical area. The system operates on the basis of reservations and therefore the origin-destination matrix for the planning period is known in advance. A penalty is imposed if the system cannot satisfy a reserved-trip. In what follows we proived a description of the system in terms of its demand and supply characteristics.

3.1 System Characteristics

i) Regions: The study area is divided into regions. The demand is assumed to be generated at the center of the regions and end up either at the same or other regions.

ii) Atoms: Each region contains a number of atoms with known population. The atoms are used to model the population coverage of the car-sharing system. In our model, we assume that there is a maximum distance that determines if an atom is covered. Thus, if there is an operating station closer than that predefined maximum value, the atom is covered.

iii) Stations: Vehicles are picked-up and dropped-off at designated stations. Stations have the necessary infrastructure for parking and for recharging the vehicles. Each station provides a specific number of parking spaces which define the station size. The station size can vary among stations and has a maximum capacity. The cost of opening and operating a station depends on the station size and location. A station can serve a trip as a pick-up (drop-off) location if the trip originates and terminates within the catchment area of the station.

iv) Vehicles: A homogeneous fleet of electric cars is used to provide the services. This means that any type of trip request can be accommodated by any available car. For each car there is an acquisition (depreciation cost) and a maintenance cost which is function of its utilization.

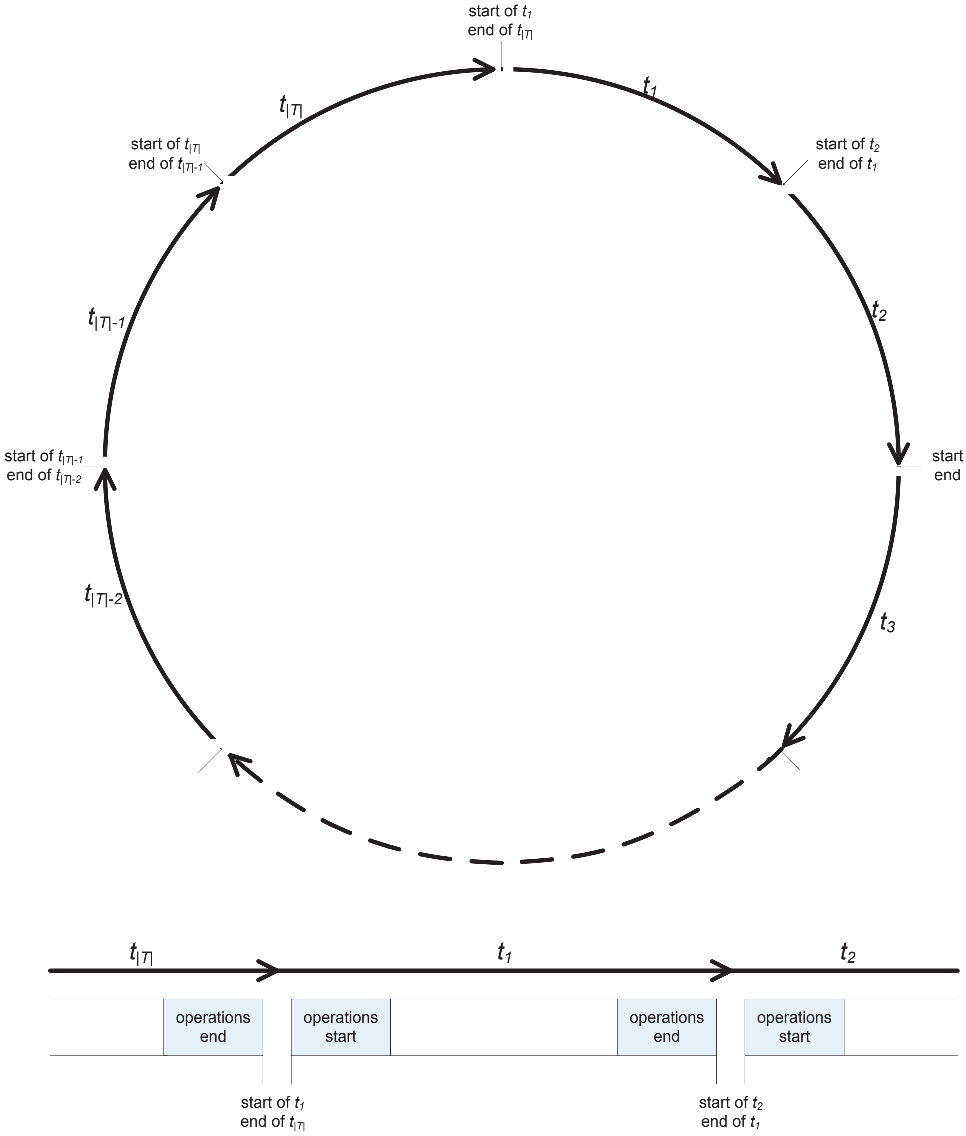


Figure 1: The relationship between time intervals and operations

v) Time Intervals: An operating day is divided into time intervals (not necessarily equal) and each operation starts at the beginning or end of a time interval. The model assumes that demand is repeated on a daily basis and the first time interval of a given day starts after the last time interval of the previous day (Figure 1)

vi) Working Shifts: A set of consecutive time intervals composes a working shift. Working shifts are used to model the man power needed for relocation operations.

vii) Demand: A demand is expressed as number of trips requested from a region to either the same or another region (not station). Each demand has also departure and arrival time intervals. In order to satisfy a trip (i) a vehicle from a station that is accessible from the origin region at the beginning of the departure time interval and (ii) a parking space from a station that is accessible from the destination region at the end of the arrival time interval have to be available. Please note that, trips do not have to be assigned to the closest stations. However, the accessibility cost forces the model to use closest region-station pairs if possible.

viii) Operations: The system contains two types of operations: rental and relocation.

a) Rental: The system operates on the basis of reservations and allows one way rental of cars. Reservations are made in advance of the pick-up period with the knowledge of origin and destination regions and pick-up and drop-off times. Cars are picked-up and dropped-off from/at a station that is accessible to the initial origin/destination region of the respective user at pre-specified (when reservation is made) periods. Each user encounters a cost for accessing/leaving the stations. It is assumed that each rental starts at the beginning of a time interval and ends up at the end of the same or another time interval (Figure 1).

b) Relocation: The system allows one way rental of cars. As a result, there might be accumulation and shortage of cars in stations. Relocation is used to rebalance the system resources, i.e. vehicles. Relocations can last more than one time interval based on the duration of the activity (Figure 1). During relocation the vehicle is not available.Total time spend for relocation in a time interval cannot be more than the total time of the man power available in that working shift.

ix) Costs: The model contains both user and operator costs.

a) User Costs: Composed of accessibility cost for each satisfied trip which contains value of time to reach from origin region to origin station and destination station to destination region.

b) System Costs: Contains unserved customer cost, vehicle operating cost, station opening cost and relocation cost. Unserved customer cost is used to penalize trip requests that cannot be satisfied. Vehicle operating cost is incurred per vehicle per day. Station opening cost has two components: fixed and variable. Fixed cost incurs if the candidate station is operating, i.e. the station is open, whereas variable cost is a function of number of parking spaces attached to the operating station. Relocation cost has also two components. The relocation personnel cost (per shift) and vehicle moving cost between stations.

x) Revenue: When a demand is fulfilled, a predefined value that depends on origin and destination stations, and starting and ending time intervals of that demand is added to the revenue.

3.2 Mathematical Model

The description of the mathematical model requires the definition of the following

sets and indices:

$i, k \in I$	region indices
$j, l \in J$	(candidate) station indices
$a \in A$	atom index
$f \in F$	working shift index
$t, u, w \in T$	time interval indices

functions:

next(t)	the time interval that is after time interval t
cover(a)	the set of stations that are accessible from atom a
btwn(t, u)	the set of time intervals between time intervals (including) t and u

parameters:

FIX_j $\left(\mathrm{VAR}_j\right)$	fixed (variable) facility opening cost for station j
VOC	vehicle operating cost per vehicle per day
MVC^t_{jl}	relocation cost of moving vehicle from station j to l starting at time interval t
CPP_f	cost of a relocation personnel for working shift f
REV^{tu}_{jl}	revenue made when a vehicle is rented at time interval t from station j to reach station l at time time interval u
CAP_j	maximum available parking space capacity for station j
OD^{tu}_{ik}	number of trips started at the beginning of time interval t from region j which ends at the end of time interval u in reach region k
LOST^{tu}_{ik}	unserved customer cost for each trip of OD^{tu}_{ik}
ACC^t_{ij} $\left(\overline{\mathrm{ACC}}^t_{ij}\right)$	accessibility cost from (to) region i to (from) station j at time interval t
COV	minimum percent of population that is assumed to be covered
MAD (MCD)	maximum accessibility (coverage) distance between regions (atoms) and stations
PR_a	ratio of population inhabited in atom a
REL^t_{jl}	time intervals needed to relocate a vehicle from station j to l starting at the beginning of time interval t
DIS^t_{jl}	the end of the time interval that the relocation is completed if a vehicle is reallocated from station j to l starting at the beginning time interval t
SHIFT_f	time intervals included in working shift f

variables:

x_j	binary variable showing if (candidate) station j is operating or not
$n^t_j \left(\overline{n}^t_j\right)$	number of available (empty) vehicles (spaces) in station j at the beginning (end) of interval t
p^t_{ij} $\left(\overline{p}^t_{ij}\right)$	number of cars rented (left) from (to) station j at the beginning (end) of period t to (from) region i
v^t	total time occupation for relocation during time interval t

$n_j^\star$	: number of parking spaces operating in station j
y_{ikjl}^{tu}	: number of vehicles assigned to demand from region i renting vehicle from station j at the beginning of interval t to reach region k through station l at the end of time interval u
z_{jl}^{tu}	: number of vehicles rented from station j at the beginning of time interval t to reach station l at the end of time interval u
m_{ik}^{tu}	: number of unserved trips of OD_{ik}^{tu}
r_{jl}^t	: number of vehicles relocated from station j to station l starting from the beginning of time interval t
$q_j^t\ \left(\overline{q}_j^t\right)$	: number of vehicles rented (left) from (to) station j at the beginning (end) of time interval t
s_f	: number of relocation personnel needed for relocation during shift f
b_t	: number of busy vehicles under rental during time interval t that are not started to be rented at the beginning time interval t
e_t	: number of busy vehicles under relocation during time interval t that are not started to be relocated at the beginning time interval t
c_a	: binary variable showing if atom a is covered by a station or not

The mathematical model can be formulated as follows:

$$\max \overbrace{\sum_{j,l,t,u} \mathrm{REV}_{jl}^{tu} z_{jl}^{tu}}^{\text{total revenue}} - \overbrace{\sum_{i,k,t,u} \mathrm{LOST}_{ik}^{tu} m_{ik}^{tu}}^{\text{unserved customer cost}} - \overbrace{\sum_{(i,j,t)} \left(\mathrm{ACC}_{ij}^t p_{ij}^t + \overline{\mathrm{ACC}}_{ij}^t \overline{p}_{ij}^t \right)}^{\text{accesibility cost}} - \overbrace{\mathrm{VOC}N}^{\text{depreciation}}$$

$$- \underbrace{\left(\sum_j \mathrm{FIX}_j x_j + \mathrm{VAR}_j n_j^\star \right)}_{\text{station opening cost}} - \underbrace{\left(\sum_{(j,l,t)} \mathrm{MVC}_{jl}^t r_{jl}^t + \sum_f \mathrm{CPP}_f s_f \right)}_{\text{relocation cost}} \qquad (1)$$

$$\text{s.t. } n_j^\star \leq \mathrm{CAP}_j x_j \quad \textbf{(a)} \qquad n_j^t \leq \mathrm{CAP}_j x_j \quad \textbf{(b)} \qquad \overline{n}_j^t \leq \mathrm{CAP}_j x_j \quad \textbf{(c)} \qquad \forall j, t \qquad (2)$$

$$n_j^\star \geq x_j \quad \textbf{(a)} \qquad \sum_t \left(n_j^t + \overline{n}_j^t \right) \geq x_j \quad \textbf{(b)} \qquad \forall j \qquad (3)$$

$$c_a \leq \sum_{j \in \mathrm{cover}(a)} x_j \quad \textbf{(a)} \qquad \sum_a \mathrm{PR}_a c_a \geq \mathrm{COV} \quad \textbf{(b)} \qquad \forall a \qquad (4)$$

$$\sum_{(j,l)} y_{ikjl}^{tu} + m_{ik}^{tu} = \mathrm{OD}_{ik}^{tu} \qquad \forall i, k, t, u \qquad (5)$$

$$\sum_{(i,k)} y_{ikjl}^{tu} = z_{jl}^{tu} \qquad \forall j, l, t, u \qquad (6)$$

$$\sum_{(k,l,u)} y_{ikjl}^{tu} = p_{ij}^t \quad \textbf{(a)} \qquad \sum_{(k,l,u)} y_{kilj}^{ut} = \overline{p_{ij}^t} \quad \textbf{(b)} \qquad \forall i, j, t \qquad (7)$$

$$\sum_{(i,k,l,u)} y_{ikjl}^{tu} = q_j^t \quad \textbf{(a)} \qquad \sum_{(i,k,j,t)} y_{ikjl}^{tu} = \overline{q}_l^u \quad \textbf{(b)} \qquad \forall l, u \qquad (8)$$

$$q_j^t + \sum_{j} r_{jl}^t \leq n_j^t \quad \textbf{(a)} \qquad \overline{q}_j^t + \sum_{\substack{l,u \\ \text{s.t. } \mathrm{DIS}_{lj}^u = t}} r_{lj}^u \leq \overline{n}_j^t \quad \textbf{(b)} \qquad \forall j, t \tag{9}$$

$$n_j^t + \overline{n}_j^{\mathrm{prev}(t)} \leq n_j^\star \qquad \forall j, t \tag{10}$$

$$n_j^t - q_j^t + \overline{q}_j^t - \sum_{l} r_{jl}^t + \sum_{\substack{(l,u) \\ \text{s.t. } \mathrm{DIS}_{jl}^u = t}} r_{jl}^u = n_j^{\mathrm{next}(t)} \qquad \forall j, t \tag{11}$$

$$\sum_{\substack{(j,l,u,w) \\ \text{s.t. } t \neq u \\ t \in \mathrm{btwn}(u,w)}} z_{jl}^{uw} = b^t \quad \textbf{(a)} \qquad \sum_{\substack{(j,l,u) \\ \text{s.t. } t \neq u \\ t \in \mathrm{btwn}\left(u, \mathrm{DIS}_{jl}^t\right)}} r_{jl}^u = e^t \quad \textbf{(b)} \qquad \forall t \tag{12}$$

$$\sum_{j} n_j^t + b^t + e^t \leq N \qquad \forall t \tag{13}$$

$$\sum_{\substack{j,l,u \\ \text{s.t. } t \in \mathrm{REL}_{jl}^u}} r_{jl}^u = v^t \quad \textbf{(a)} \qquad \sum_{t \in \mathrm{SHIFT}_f} v^t \leq s_f \quad \textbf{(b)} \qquad \forall t \text{ and } \forall f \tag{14}$$

$$x_j, c_a \in \{0, 1\} \qquad \forall j, a \tag{15}$$

$$n_j^t, \overline{n}_j^t, n_j^\star, p_{ij}^t, \overline{p}_{ij}^t, r_{jl}^t, m_{ik}^{tu}, z_{jl}^{tu}, y_{ikjl}^{tu}, q_j^t, \overline{q}_j^t, b^t, e^t, v^t, s_f, N \in \mathcal{N} \qquad \forall i, k, j, l, t, u, f \tag{16}$$

Equation 1, is the objective function of the model and seeks to maximize the profit of the operator. Equation 1, includes six terms. The first term expresses the total revenues resulting from the operation of the system. The second term refers to the cost incurred by the operator due to lost customers , i.e. customers that cannot be served due to the unavailability of cars. The third term expresses the accessibility cost encountered by the users of the system. The accessibility cost depends on the number of the car-sharing stations that will be open. The lower the number of stations the higher the time needed to reach them and consequently the higher the accessibility cost. The fourth term represents the depreciation cost of the vehicles and depends on the fleet size. The larger the fleet size the higher the vehicle depreciation cost will be. The fifth and sixth terms relate to the station opening and car relocation costs respectively. The station opening cost and the car relocation costs are increasing as the number of open stations and the car relocation operations are increasing.

The objective function of the model also addresses, implicitly, the trade-off between the operator's profit and the level of service offered to the system users. The penalty cost for unserved customers is increasing (second term of the objective function) as cars are not available at the right place at the right time, i.e. where and when the customers require them. The unavailability of cars at the right place the right time is influenced by the fleet size and the number of car relocations. The smaller the fleet size and the number of car relocation operations the larger the penalty cost for unserved customers will be and the smaller the operator. On the other hand an increase of the fleet size and car relocation operations results to an increase of the operating costs and consequently to a reduction of the profit of the operator. The inclusion of the accessibility cost in the objective function also contributes to the examination of the trade-off between revenues and the level of service. The accessibility cost of the users is increasing as the number of open stations is decreasing.

The above discussion suggests that a lower commitment of resources in terms of number of stations, fleet size, and car relocation operations leads to a reduction of the cost of the operator. On the other hand the inclusion of the users' accessibility cost and the penalty cost for lost customers leads to an increase of the resources made available by the operator and consequently to an increase of the level of service offered to the system users. Constraints 2a, 2b, 2c restrict the number of parking spaces (station capacity constraint), and the number of available vehicles and for each time interval and station. If a station is not open in a candidate station location the station capacity is set equal to zero. If the station is open then there is an upper bound (CAP_j) for its capacity. Constraints 3a, 3b require that at least one parking space and pick-up and drop-off location is assigned to a rented vehicle. These constraints are essential in order to guarantee the coverage of the demand by an open station that has at least a capacity of one parking space. Constraints 4a , 4b are the atom coverage constraints, i.e. if an atom covered or not, and population coverage constraints, i.e. the shared-use system is accessible by a given percentage of the population, respectively. Constraints 5, ensure that the total demand is equal to the sum of the satisfied and unserved (lost) demand.

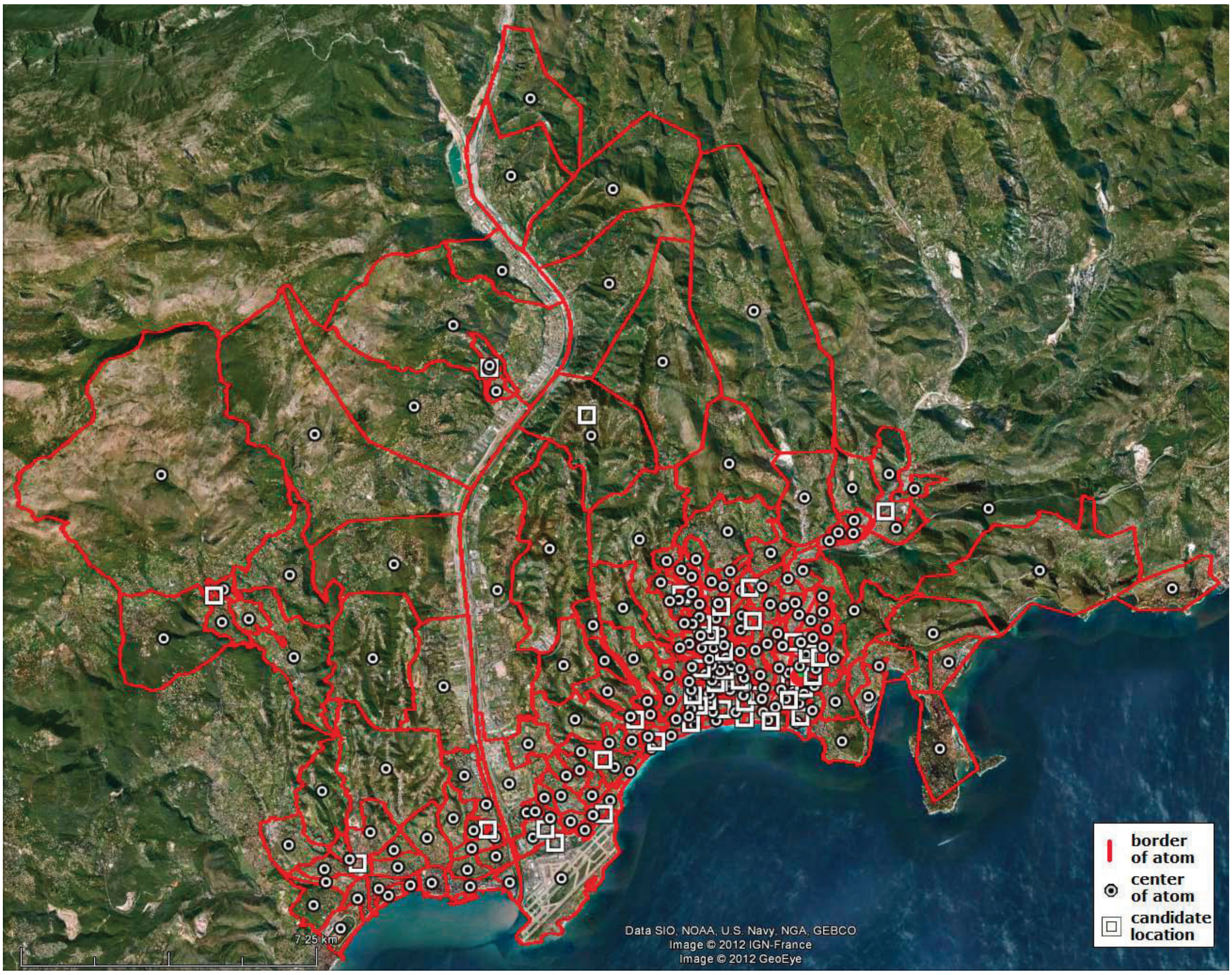
Constraints 6, postulate that the total number of vehicles assigned to each station j to cover the demand originating from all regions i at the beginning of interval t to reach destination regions k through station l at the end of interval u, is equal to the number of vehicles rented from station j at the beginning of interval t to reach station l at the end of interval u. Constraints 7a, indicates that the total number of vehicles assigned to each station j to cover the demand from regions i for all destination regions k through stations l at the end of all intervals u is equal to the number of vehicles rented from station j at the beginning of time interval t to serve demand from regions i. Constraint 7b does the same as constraint 7a for the cars originating from region i left at station j at the end of period t. Constraints 8a and 8b are equivalent of constraints 7a and 7b and ensure respectively the same conditions for the cars that are relocated to/from a station j. Thus, constraints 6, 7a, 7b, 8a and 8b, establish the functional relationship between the variables y, z, p ($\overline{p}$), and q ($\overline{q}$) respectively. Please note that, variables z express car assignments independent of the region to which originate/end their movement, variables p and $\overline{p}$ indicate customer movements from regions to stations and from stations to regions respectively, and variables q and $\overline{q}$, signify region station and station region assignments respectively.

Constraints 9a require that the number of cars leaving a station (due to rental and relocation) at the beginning of interval t cannot exceed the number of vehicles available at that stations at the same time interval. Constraints 9b suggest that the number of cars entering a station (due to rental drop-off and relocation) cannot be more than the number of available parking spaces at the same time period. Similarly, constraints 10 force the total number of parking spaces at the station minus the number of empty parking spaces at the end of a time interval should be at least equal to the number of cars present at the station at the beginning of the next time interval. Constraints 10, require that the total number of parking spaces available at a station at the end of a time interval plus the available number of cars at the beginning of the next time interval is equal to the number of parking spaces existing in a station. Constraints 11 are the "car conservation" constraints for each station. Constraints 12 are used to establish the functional relationship between variables b, e, and z, r respectively. Variables b and e are used in constraints 13 which are used to determine the total number of cars (fleet size) used in the system. Constraints 14a, 14b are introduced to ensure the per shift availability of workforce needed to perform car relocations. Constraints 14a assign the number of relocation operations needed at each time interval , while constraints 14b ensures that the number of persons needed to perform the relocation operations per shift will not exceed the available workforce per shift.

4 Model Application

The model presented in the previous section was applied to plan a station based shared-use electric vehicle system in Nice, France. The study area is 294.19km^2, and has a population 327188 inhabitants between ages 15-64, with a density 1112 persons/km^2. The area under consideration consists of 210 regions. The population of each region has been obtained from 2009 census data. The demand for shared-use services originates from each region and is assumed to be known for each time interval. In total 42 locations were considered to be candidate locations for establishing a shared-use station. The maximum capacity of each candidate station location was established to be 5 car parking spaces. Furthermore, the distance used to determine if the population of an atom is considered to be covered was defined to be 500 meters in the base case scenario. Data related to the cost of opening a station, car operating cost, cost of relocation operations, penalty costs for lost customers, station accessibility costs, and unit costs for estimating system revenues were obtained from the company (Veolia) operating the Nice system and from relevant literature sources (de Almeida Correia and Antunes, 2012, Cepolina and Farina, 2012).

Figure 2a illustrates the study area and its partition into regions and the candidate station locations, while Figure 2b summarizes the values of all the parameters needed to run the base model scenario and the associated sources of information. In all the other scenarios, all the parameters that are not specifically mentioned are set to these values.



vehicle operating cost (€/day):	17	lost trip cost (€/trip):	20
station opening cost (€/station/day):	30	station capacity (veh):	5
parking space operating cost (€/space/day):	5	accessibility distance (km):	0.5
minimum accessible population:	25 %	accessibility cost (€/km):	5
revenue per time interval (€/int):	8	revenue per distance (€/km):	0.1
relocation cost (€/km):	0.12	relocation speed (km/h):	30
personnel cost per hour (€/personnel):	12	relocation coefficient:	4
length of each working shift (h):	8	number of time intervals	15

Figure 2: (a) The study area in Nice, France and (b) the parameters used in the mathematical model

The results of the solution of the proposed model provide useful information to decision makers regarding:

- The location and number of car-sharing stations and parking spaces per station
- The service regions of the car-sharing system
- The required fleet size and the initial allocation of cars to stations

- The total workload associated with car relocation operations and the workforce per shift
- The average car-utilization
- The average and the per station utilization of the parking spaces
- The total costs associated with the users (station accessibility) and the operator (car operation, unserved customers, establishment and operation of stations)
- Operator revenues and profit

One of the advantages of the proposed model is its ability to create a large number of alternative scenarios that can be used to study:

- The trade-off between operator's profit and accessibility cost, and population coverage.
- The sensitivity of the solution in terms of the values of such parameters as unit vehicle operating cost, unit labor cost of relocation personnel, and penalty cost for unserved customers.

IBM ILOG Cplex version 12.2 was used to solve the proposed model. The resulting application was run on a Intel Core2 Quad 3.00 Ghz and the resulting CPU times were limited to 3600 seconds and the absolute optimality gap is limited to 0.5 %. The results of the different scenarios of the proposed model are summarized in Figure 3. In the base scenario a demand of 200 one-way trips are used. Since the current system of Veolia is a two-way shared-car system, we have divided each trip into subtrips. The registered trips are divided into subtrips if the vehicle does not change its location for more than 60 minutes. In addition to that, we have filtered the trips used in the problems in such a way that, there is at least a candidate station that is 300 meters close to both origin and destination region of that trip.

A base scenario was analyzed with values for the parameters as in Figure 1. The different user and operator costs associated with the model are summarized in the part of Figure 3, which describes the base scenario. Additionally, average car utilization rate is 53%, while the total net benefit is 2231.46 €.

Furthermore, we perform a sensitivity analysis (SA) for the demand increase. We consider the demand level is critical for the efficient operation of the system and it can provide useful insights to the operator regarding the level of service. For each different scenario an optimization is performed and values for the fleet size, the number of stations and the number of total parking spaces is also shown in Figure 3.

Figure 3 shows the effect of demand increase. In this graph, we compare the demand increase on two scenarios. In the "fixed spaces" scenario, the problem is solved for 200 trip requests, selected stations and their sizes are fixed and number of requests is increased to 250, 300, 350 and 400 trips. In the other scenario, namely "unrestricted", operating stations and parking

number of vehicles:	64	number of stations:	21
number of parking spaces:	88	ave. number of parking spaces per station:	4.19
ave. car utilization (hour based):	53%	ave. car utilization (interval based):	62%
number of relocations:	54	distance traveled in relocations (km):	105.48
number of relocation personnel:	2	ave. number of relocations per car:	0.84
number of served trips:	169	number of lost trips:	31
accessibility cost (€):	407.52	unserved customer cost (€):	620
station opening cost (€):	1070	relocation cost (€):	204.66
vehicle operating cost (€):	1088	total cost (€):	3390.18
revenue (time intervals) (€):	4936	revenue (distance) (€):	685.64
total revenue (€):	5621.64	total profit (€):	2231.46

Table 1: Output values for the base scenario

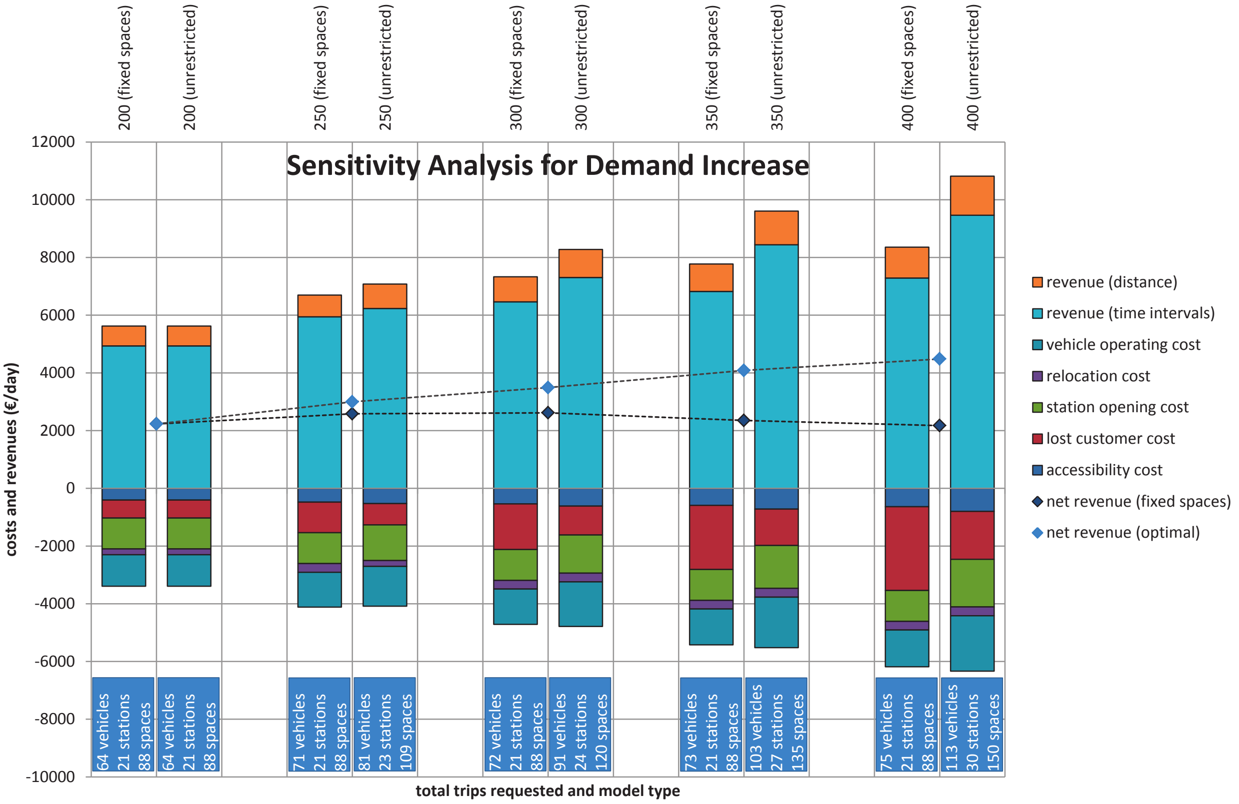


Figure 3: Computational results for different demand levels

spaces are not limited and the "unrestricted" problem is optimally solved for the same requested trip set. Figure 3a shows that, as demand increases, the profit difference of the systems increases between two models. In addition to that, the number of vehicles is also increasing in order to fulfill the requested trips in both scenarios. However, the increase is slow in the "fixed spaces" scenario whereas it is faster in the "unrestricted". Last but not least, readers can also observe that although the revenue from served trips increased in both cases as demand increases, the net system profit of the "fixed spaces" scenario decreases. This is the outcome of the increase in the number of unserved trips. This means that while a small increase in demand can be absorbed by the existing system without new infrastructure, a larger increase would require a significant increase in the resources and possibly the opening of new stations to continue offering a high quality service. Note the higher number of lost customers with the increase of the demand in the "fixed spaces" scenario.

5 Concluding Remarks

A model for supporting strategic and tactical planning decisions for car-sharing systems was developed and tested in a large scale real world setting. The proposed model closes a gap in the existing literature by considering simultaneously decisions associated with the allocation of strategic assets, i.e. stations and vehicles of car-sharing systems and the allocation of personnel for relocation operations (tactical decision). The model provides the decision makers with ample opportunities to perform sensitivity analysis for the relevant model parameters, a feature particularly useful for cost values that are difficult to establish empirically, e.g. unit cost of unserved customers, population coverage, station accessibility cost. Furthermore, the model allows the decision maker to examine the trade-off between profit and the level of service offered to the public. This last feature is of particular importance if we consider that car-sharing systems are subsidized with public funds. The results obtained from the application of the model to a case resembling real world decision making requirements, give satisfactory results.

Research work under way involves the development of a multi-criteria formulation having similar features with the proposed model which will allow the explicit consideration of the preferences of the decision maker in terms of the relevant importance of the operator's profit and the level of service offered to the public. Another stream of research that will extend and enhance the work presented in this paper is the integration of the proposed model with a simulation model that will provide a more realistic representation of the relocation operation costs. In the proposed model, the system is only optimized for a single scenario. We plan to alter the model in such a way to be capable to optimize the system for multiple demand scenarios on a single configuration. This will create a more robust setting that will integrate stochasticity and fluctuations in the demand. Modeling the operational problem and assigning the vehicle rosters while taking their electrical charge level into consideration is another future work directions of this project.

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