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Abstract

The introduction of Intelligent Transportation Systems (ITS) technologies and new sensing hardware promise significant progress in reducing the congestion level in cities. Recent advances in the GPS technology and the probe vehicles deployment offer an innovative prospect for arterial travel time research. Specifically, we focus on estimation of arterial route travel time distribution which contains more information in regard to arterial performance measurement and travel time reliability. Speed of vehicles at a given time in the network is not a deterministic quantity over space because of drivers' behaviours (conservative vs. aggressive drivers), the spatial effect of signals (near the signal line vs. further upstream) and temporal-spatial pockets, where average speed is temporarily different than the widespread average (e.g. point bottleneck in a freeway system). In this work, we will analyse the distribution of travel time for routes (expressed as series of links) for a period comparable to a signal cycle. In that way we smooth the variations of traffic for a specific link because of the time-dependent capacity of a signal and the variability in driver's behaviour as many vehicles pass over a link in that period.

In the proposed technique, probe vehicles provide travel time of the traversing links. For each two consecutive links, a two-dimensional (2D) diagram is established so that data points represent travel times of a probe vehicle crossing both two aforementioned links. States in the 2D diagrams (defined as rectangular clusters) consist of data with homogenous travel times. The state boundaries can be set by basic travel time rules such as free flow travel time and oversaturated conditions. In addition, a heuristic grid clustering optimization method is done to determine boundaries in order to have more analogous states. By applying markov chain procedure, we relate states of 2D diagrams to the following one; compute the transition probabilities, and partial travel time distributions to obtain the arterial route travel time distribution. The procedure with various probe vehicles sample size is tested on an arterial Lincoln Blvd., Los Angeles, CA, during morning peak, which has been simulated in a microscopic simulator. The simulated results are very close to the markov chain procedure and more accurate once compared to the convolution of links travel time distributions. The promising results capture the fundamental characteristic of field measurements.

Keywords

Grid clustering – Markov chain – Probe vehicle – Travel time distribution - Travel time variability

1. Introduction

Nowadays, congestion is a widespread time consuming phenomenon in urban areas and the key step to alleviate it is traffic network observation and data collection. Hence, traffic monitoring is a crucial component in management of transportation systems for traffic control and guidance purposes. The integration of Global Positioning System (GPS) technology within the ITS framework introduces a new paradigm in traffic surveillance: probe vehicles. Despite to old-fashioned traffic sensors, inductance loop detectors, probe vehicles offer further information like vehicle trajectory in more convenient manner. In principle, a steadily incremental public deployment rate of GPS-equipped vehicles, low maintenance cost, and inherent distributed characteristics lead to tackling probe vehicle challenges in traffic monitoring research such as travel time estimation.

Travel time is a crucial index in assessing the operation efficiency of traffic network. It establishes a common applicable layer among all the perspectives of individual travellers and practitioners. In addition, it can be an indicator of congestion level of transport network when it is compared to the free flow travel time. There is an intrinsic difference between characteristic of travel time in urban networks and freeways. Interrupted urban traffic flow because of traffic signals, driveways, side-parking effect, and right and left turning movements increase the variability in travel time and make the estimation problem more intriguing. Additionally due to the size of urban systems, traffic monitoring and travel time estimation in urban arterials has fallen behind once compared with freeway developments.

Travel time variability designates the variation of various trip travel times over a specific path. Travel time variability can be investigated from several point of views [1]: vehicle-to-vehicle variability which is correspond to different vehicles traveling the same route at the same time, period-to-period variability corresponding to different travel times of vehicles traveling the same route at different periods within a day, and day-to-day variability addressing the travel time variations of vehicles crossing the same route at the same period of time on different days. Variation in transport network demand and supply profiles is the main source of travel time variability. Non-recurrent and peculiar events, such as incidents, lane closures, and sport events can also cause significant travel times deviations from recurrent conditions. Reduction in travel time variability is at least as desirable as reduction in mean travel time in regard to road users since it decreases commuting stress and transportation mode and route choice decision making uncertainty.

There is a vast literature, which addresses different travel time estimation approaches state to different applications and terms. Recently an analytical model was developed to estimate the travel times on arterial streets based on data commonly provided by loop detectors system and the signal settings at each traffic signal [2]. The model considers the spatial and temporal queuing at the traffic signals and the signal coordination to estimate travel time in arterials. The model implementation is straightforward and unlike other approaches does not depend on site specific parameters or short term traffic flow predictions that make transferability to other locations very difficult. Several extensions and enhancements were developed and implemented to the analytical model explicitly address the issues of long queues and spillovers that frequently occur on arterials in urban areas [3]. The model has also been integrated into a pilot arterial performance measurement system in California.

The aforementioned studies, focused on average travel time estimation of travelling vehicles in one cycle. Nevertheless, statistical scalar indexes (e.g. mean, variance, percentiles, and etc.) are not fully enlightening about travel time variability compare to travel time distribution (TTD). To address the uncertainty issue, a new trend is seen in recent travel time research. For instance in [4], a multistate model is employed to fit a mixture of normal distributions into travel time observations of an expressway corridor. Each normal distribution is associated with an underlying traffic state providing quantitative uncertainty evaluation. The multistate mixture models results in better fitting, revealing that TTD usually has more than one mode which is totally dependent on time horizon of study, demand, topology, and etc.

Uno *et al.* discuss route travel time variability using bus probe data in [5]. In pre-processing stage of their method, map matching procedure and a data filtering for dwell time elimination are done. Afterwards, they decomposed a route to sections with lognormal TTD to estimate the bus route TTD. Finally, they used the coefficient of variation and 85 and 50 percentiles of average travel time to evaluate the travel time reliability. In [6] a statistical evaluation is investigated trying to assess feasibility of probe vehicles employment for collecting traffic information. The authors analytically answered to few fundamental questions about data sampling and reporting rates and probe vehicles penetration level in a single road link without any validation with field or simulation data. They assume a binomial distribution for number of probe vehicles and a Poisson distribution for reporting rate, deriving formulas for mean and variance of reports number and speed estimation, and confident reporting intervals.

Delay at signalized intersections is the main source of uncertainty in TTD which is tackled in [7]. Zheng *et al.* propose an analytical method for estimation of urban link delay distribution. The results indicate a correlation between arrival time and link travel time under different degree of congestion. Their model demonstrates evolutions of delay distribution as well, so that both average and variance of delay increase cycle by cycle. Ultimately, the delay uncertainty is assessed by the distance between 90 and 10 percentiles divided by the 50 percentile (median) of delay. Sparse probe vehicles is discussed in [8] for arterial traffic estimation and short term prediction of travel time. The authors proposed a statistical modeling framework that captures the evolution of traffic flow as a Coupled Hidden Markov Model. An expectation maximization algorithm is developed to adopt the model parameters. Their methodology is solely based on two assumptions: independence of link travel time from other traffic variables and independence of state transitions from all other current and past link states. The evaluation is done using dataset from a taxis fleet in San Francisco, CA, as a part of Mobile Millennium project.

In this paper, we estimate the probability distribution of arterial route travel time given probe vehicles link travel times. The input to the procedure is probe vehicles travel times of all links that are crossed and belongs to the route; and the output of procedure is TTD over the study time horizon. The developed procedure is based on Markov chain to address both the traffic progression and correlation between links. The chief struggle with utilizing probe data is that travel time of a probe vehicle is principally a sample of a random variable (travel time). This raises inquiries about the probe vehicles penetration rate required for producing a proper sample set. Ultimately, the procedure is studied on Lincoln Blvd., Los Angeles, CA. simulated in AIMSUN microscopic simulator.

The outline of the reminder is as follows. In section 2, we briefly discuss about the Markov chain procedure, its application in traffic engineering, and via an example we illustrate our motivation. Then, we introduce our proposed methodology in section 3. The study site, data and simulation details; results and discussion are presented in section 4 and 5, respectively. Finally in section 6, conclusions are drawn and future works are summarized.

2. Motivation

The straight forward method for route TTD estimation while having individual link TTD is to aggregate those independently [9]. Assume a route consisted of K links with signalized intersections. The route TTD is computed according to (1), where the (*) mathematical operator expresses convolution:

$$TTD_K = TTD_1 * TTD_2 * \dots * TTD_k \tag{1}$$

Evidently, the above method considers independence (no spatial and temporal correlation) between single TTDs. Consequently, lots of spatial-temporal correlation information is neglected.

To be more illustrative, figure 1 provides an example to investigate the effect of correlation in route travel time. Imagine a route consist of two serial links. Part *(a)* depicts a 2D diagram in order that each point denotes link 1 and 2 travel times of a probe vehicle. In this hypothetical case, it can infer that vehicles which are fast in link 1 (without delay) are also fast in link 2 (left-lower part). In contrast, some vehicles are slow in both links (right-upper part). Given the link TTDs in *(b)*, if the convolution method is utilized the convolved route TTD will be totally different from real TTD. Although the real TTD is bi-modal (revealing there are two groups of vehicles, slow and fast) with a distinct zero flat space between peaks, the convolved TTD has three peaks.

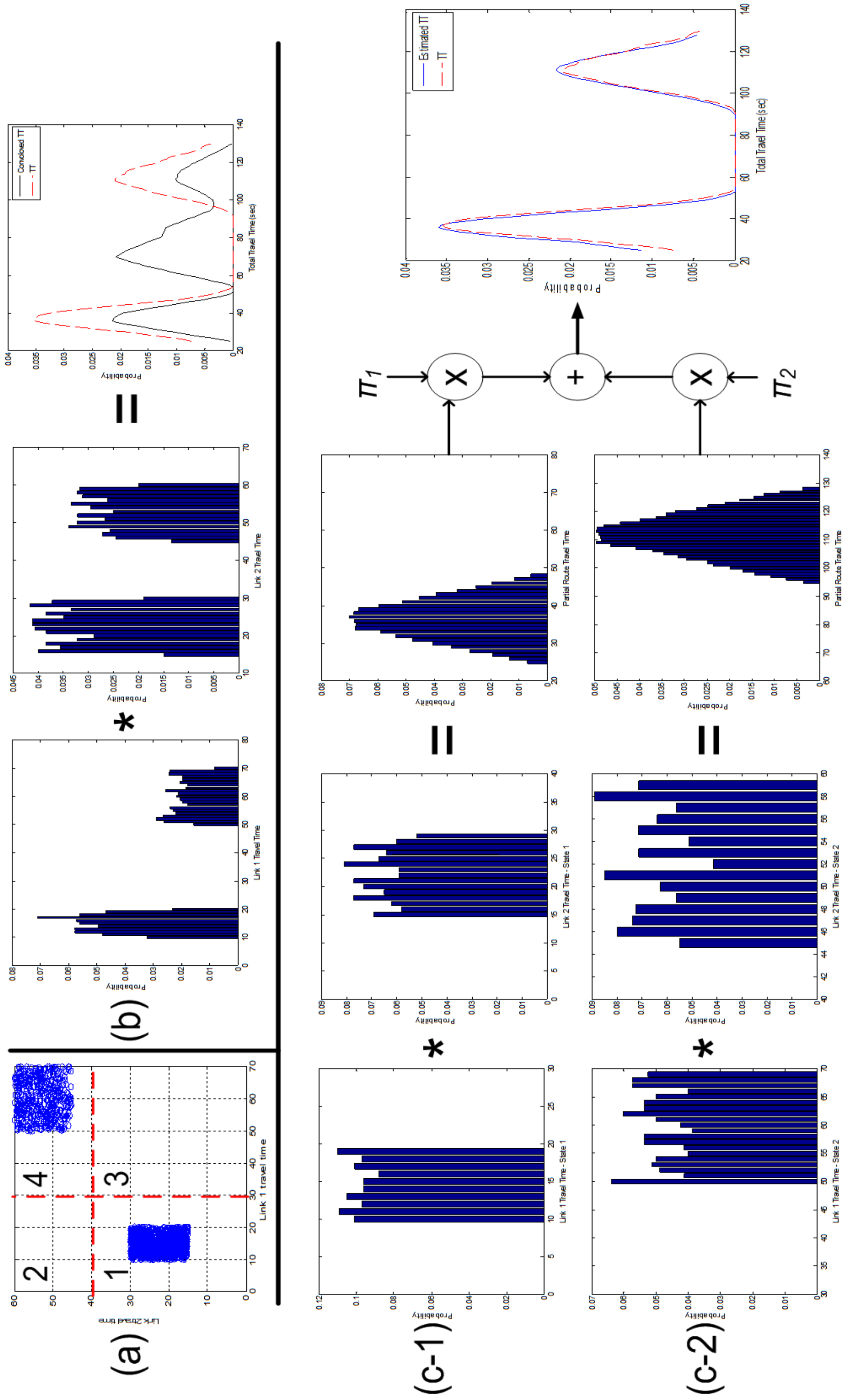
To capture correlation patterns between link travel times, one can divide travel times of one link to different states. In figure 1-a, 2 states (slow and fast) are defined for each link. The initial value (π) of each of the states in link 1 and the probabilities of transitions (P) between states of link 1 and states of link 2 as traffic progress are according to:

$$\pi = \begin{bmatrix} P_{fast} \\ P_{slow} \end{bmatrix} = \begin{bmatrix} \frac{N(1)+N(2)}{N(1)+N(2)+N(3)+N(4)} \\ \frac{N(3)+N(4)}{N(1)+N(2)+N(3)+N(4)} \end{bmatrix} = \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix} = \begin{bmatrix} 0.625 \\ 0.375 \end{bmatrix} \tag{2}$$

$$P = \begin{bmatrix} P_{fast,fast} & P_{fast,slow} \\ P_{slow,fast} & P_{slow,slow} \end{bmatrix} = \begin{bmatrix} \frac{N(1)}{N(1)+N(2)} & \frac{N(2)}{N(1)+N(2)} \\ \frac{N(3)}{N(3)+N(4)} & \frac{N(4)}{N(3)+N(4)} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{3}$$

Where *N(i)* is the number of points inside each cluster (as shown in figure 1-a), P_{fast} is the probability of vehicles being fast in link 1, and $P_{slow,fast}$ is the corresponding probability of being slow and fast in link 1 and link 2, respectively. Given 2 states for each link, there are 4 combinations of states, hereafter named as markov path. Every one of 4 markov path has a probability of occurrence and a corresponding partial TTD which is convolution of travel times of link states. The partial TTD for markov path of fast-fast and slow-slow states are seen in figure 1-c-1 and 1-c-2, respectively. At the end of procedure, the partial TTD of markov paths is multiplied by the consistent initial and occurrence probabilities and the total sum is calculated to find the final route TTD. The final result of estimation method is closely matched with real TTD as it is apparent.

Figure 1 Explanatory example about motivation of integration of markov chain into TTD estimation



It is worth to mention that travel times of consecutive links in reality is not as well-ordered as figure 1-a. Instead, they are more similar to figure 2 which makes the state boundaries less clear and, moreover, the privilege effect of partitioning to states becomes less beneficial and less straightforward. This contamination of data led to a grid clustering problem, yet the whole concept remains the same. The above example can be applied to several links using markov chain procedure which is briefly reviewed in next sub-section. The complete version of methodology is fully presented in the next section.

2.1 Markov Chain Procedure

Markov chain is a technique for statistical modelling of a random process that the state of system changes through progression. A markov chain is entirely demonstrated with the set of state definition and transition probabilities. The transition probabilities are associated with the manner of state progression during the system evolution. A system which has the markov property satisfies the following: the conditional probability of the system being at the next state given the current state depends only on the current state and not on the previous states of the system. Mathematically put:

$$Pr\{s_{t+1} = s' | s_t, s_{t-1}, \dots, s_1\} = Pr\{s_{t+1} = s' | s_t\}. \tag{4}$$

The markov property empowers markov chain to capture both probabilistic nature of travel time and successive links travel times fundamental correlated feature. In other words, traffic progresses like a markov chain in arterials (sequence of links) which is well-matched with physics of traffic. The most intuitive situation is that the current link travel time depends on the travel time of immediate upstream link.

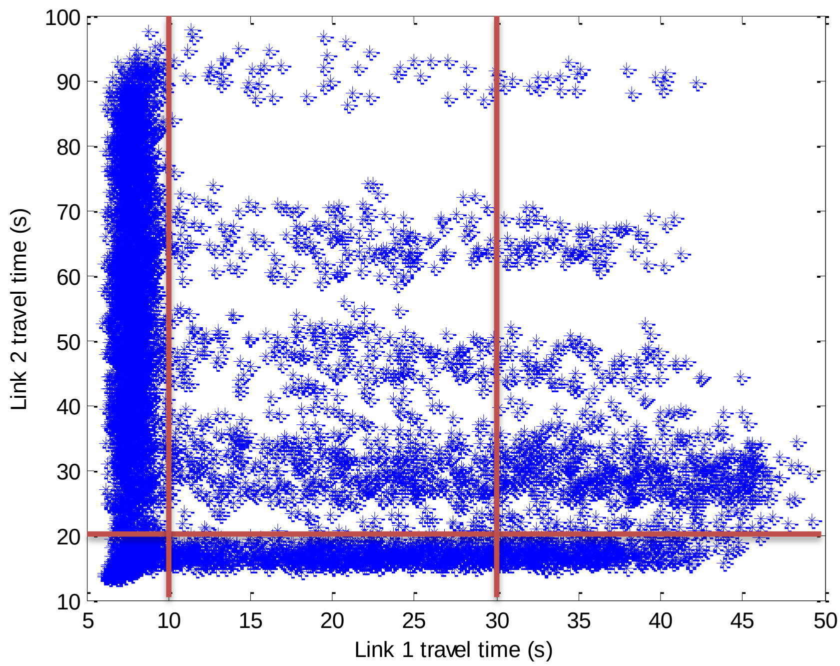
Markov chain is utilized in vast fields of transportation research. Discrete time markov chain for estimation of expected freeway travel time is investigated in [10] where states correspond to congestion level of links. The authors find the average travel time of each link both in non-congested and congested states using field data and with consideration of transition probabilities between different states, the route mean travel time is estimated. In [11] a markov chain is developed to model the effect of freeway flow breakdown and recovery in travel time reliability. States of the freeway is consisted of flow rate (divided to equal intervals) and 2 categories of speed (low and high). 5-min aggregated data of weekday peak period over one year is used for calibration of transition matrix. Geroliminis and Skabardonis also proposed an analytical model using markov chain for prediction of platoon arrival profiles and queue length considering platoon dispersion in arterials [12]. Markov decision process is used to model traffic between traffic signals and the traffic dynamics are modelled within the kinematic wave theory.

3. Methodology

In our proposed methodology, the raw measurements are individual link travel times traversed by probe vehicles. A probe vehicle path may consist of one to as many links as the study route have, which makes the number of link travel times reported by every probe vehicle different. With high resolution GPS data (position plus time stamp), finding the trajectory of moving vehicle is not a complicated task, which basically include link travel times. Note that errors in trajectories, map matching, or low resolution data are not addressed in this work.

Afterward, travel times of all probe vehicles crossing two successive links during data collection period are used to construct a 2D diagram corresponding to the two links. 2D diagrams are graphical representations of vehicles travel times joint distributions. Given a route consisting of *K* links, *K-1* 2D diagrams are established in order to identify the markov chain structure i.e. states definition and transition probabilities. Figure 2 illustrates such a diagram for link 1 and 2 consisting of 8177 samples. Note that we are interested to estimate the TTD of through movement, so 8177 samples are vehicles that crossing link 1 and 2 and then move forward to the third link. Also note that a large fraction of vehicles crosses link 1 or 2 without any delay, but the variation of travel time is large.

Figure 2 2D diagram showing the joint distribution of links 1 and 2 travel times



3.1 Markov chain identification

As a 2D diagram is constructed, states, transition probabilities, and initial state probabilities of markov chain should be identified appropriately. We consider link states as time intervals which is compatible with road user implication from travel time states. For example 3 states for link 1 travel time and 2 states for link 2 travel time are seen in figure 2. Let$\mathcal{X}_i = \{x_{i1}, \dots, x_{i(m_i-1)}\}$and $\mathcal{Y}_i = \{y_{i1}, \dots, y_{i(n_i-1)}\}$ denote sets of boundaries in 2D diagram *i* (correspond to link *i* and *i+1*), respectively. Consequently, there are m_i and n_i states for link *i* and *i+1* travel time, respectively. For instance, the first state of link *i* indicates travel times in $[\min(tt_i), x_{i1})$ and the last state represents travel times inside $[x_{im_i}, \max(tt_i)]$ interval. Note that all links except the first and last one have two different sets of states. This kind of state definition yield to rectangular regions in 2D diagrams which is used to define initial state and transition probabilities.

Initial state probabilities in markov chain are the probabilities of link 1 travel time states which are as follows:

$$\pi = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_{m_1} \end{bmatrix} = \begin{bmatrix} \frac{N(1)}{\sum_{i=1}^{m_1} N(i)} \\ \vdots \\ \frac{N(m_1)}{\sum_{i=1}^{m_1} N(i)} \end{bmatrix}, \tag{5}$$

where *N(i)* denotes number of data in state *i* of link 1 (i.e. link 1 travel times in $[x_{1(i-1)}, x_{1i})$).

To identify transition matrix between two successive links, transition probabilities should be defined**.** The generic transition matrix for link *l* is shown as the following:

$$P = \begin{bmatrix} p_{1,1} & \cdots & P_{1,m_{(l+1)}} \\ \vdots & & \vdots \\ P_{m_l,1} & \dots & P_{m_l,m_{(l+1)}} \end{bmatrix} = \begin{bmatrix} \frac{N(1,1)}{\sum_{i=1}^{m_{(l+1)}} N(1,i)} & \cdots & \frac{N(1,m_{(l+1)})}{\sum_{i=1}^{m_{(l+1)}} N(1,i)} \\ \vdots & & \vdots \\ \frac{N(m_l,1)}{\sum_{i=1}^{m_{(l+1)}} N(m_l,i)} & \dots & \frac{N(m_l,m_{(l+1)})}{\sum_{i=1}^{m_{(l+1)}} N(m_l,i)} \end{bmatrix}, \tag{6}$$

$$p_{i,j} = \Pr\{S_{l+1} = j | S_l = i\},$$

where S_l indicates state of travel time in link *l*, and *N(i,j)* is number of data within each rectangular region confined by state *i* in link *l* and state *j* in link *l+1*.

There are $m_1.n_1 \dots m_{k-1}.n_{k-1}$state combinations from origin to the destination of the route and each one of them is named as a markov path. For a given markov path, all of the transition probabilities in each step are multiplied to compute path probability (formula 7) because Markov property makes joint probability of each step in the Markov chain independent.

$$\Pr\{S_1 = i_1, S_2 = i_2, \dots, S_k = i_k\} = \pi_{i_1}\, p_{i_1,i_2} p_{i_2,i_3} \dots p_{i_{(k-1)},i_k} \tag{7}$$

Then we obtain markov path TTD using convolution of partial TTD of each link state.

$$\mathrm{TTD}\{S_1 = i_1, S_2 = i_2, \dots, S_k = i_k\} = TTD(i_1) * TTD(i_2) * \dots * TTD(i_k) \tag{8}$$

Finally, the route TTD is equal to weighted sum of markov paths TTD with path transition probabilities weights.

$$TTD = \sum_{i=1}^{m_1.n_1 \dots m_{k-1}.n_{k-1}} \Pr(markov\ path_i)\,.\,\mathrm{TTD}\,(markov\ path_i) \tag{9}$$

3.2 Clustering

The chief challenge in our methodology is to properly identify rectangular clusters (i.e. $\mathcal{X}_i$ and $\mathcal{Y}_i$) so states exhibit homogenous travel time characteristics. Intuitive thumb rules like travel time in free flow, near capacity, and oversaturated conditions may be utilized to address the state identification problem. Furthermore, heuristic grid clustering optimization methods can be applied to determine boundaries in order to have more analogous states.

3.2.1 Traffic engineering guidelines

Intuitive traffic engineering rules to estimate travel time for conditions with different level of congestion are employed to define state boundaries. For instance in uncongested situation, we consider free flow travel time as follows:

$$TT_{ff} = \left(\frac{link_{length}}{V_{ff}}\right)(1+\varepsilon), \quad \varepsilon = 0.1 \tag{10}$$

Furthermore, the second order estimate of travel time in near capacity regime is R which is the red interval for through movement. The next boundary depends on the case study. In oversaturated conditions, it can be set as cycle time, C, where vehicles are delayed for more than one cycle time; and in case of semi-congested conditions, $R+TT_{ff}$ can be considered as state limit.

3.2.2 Grid clustering

Due to the implication of travel time as time intervals, grid clustering seems a reasonable clustering approach. Although k-means and its variants have been the dominant algorithms in practice, they have the shortcoming to predetermine the number of clusters. Additionally, the clustering results are significantly influenced by cluster initialization, and shape (i.e. distance metric). The chief trait of grid clustering method is that it uses a multi-dimensional imaginary grid structure which partition the data space to hyperrectangles in order to finds the hidden patterns in data. Then as the next step, the hyperrectangles are grouped with respect to a topological-neighboring criterion and the problem attributes to produce clusters. The grid clustering has shown to be very effective for analyzing huge datasets and shows superiority performance over fuzzy k-means and RBF networks [13].

In a heuristic way, we draw a one second by one second 2D Cartesian grid over each 2D diagram and goal is to find a subset of 1 second boundaries in both axes to have uniform rectangular regions. In the grid clustering, we introduce a function that measures the discrepancy between data of two consecutive columns of grid as follows:

$$\alpha_j = \sum_{i=1}^{n} \left|\frac{x_i - y_i}{x_i + y_i}\right|, \tag{11}$$

where, α_j is the measured discrepancy of column j and $j+1$, n is number of small squares in each column, and x_i and y_i is number of data in small squares in column j and $j+1$, respectively. A similar procedure is done for rows of grid of each 2D diagram. Then according to these values, we choose a few points with highest discrepancy value (α) as state limits.

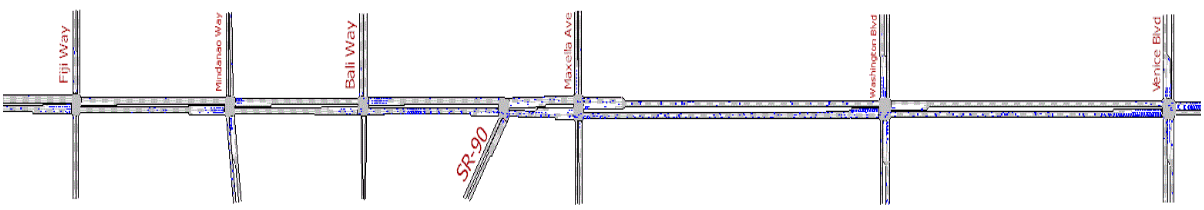
4. Study Site: Lincoln Avenue

This test site is 2.3 km long stretch of a major urban arterial with speed limit of 35 mph, north of the Los Angeles International Airport, between Fiji Way and Venice Boulevard in the cities of Los Angeles and Santa Monica. The study section includes seven signalized intersections with link lengths varying from 150 to 500 meters. The number of lanes for through traffic per link is three lanes per direction for the length of the study area. Additional lanes for turning movements are provided at intersection approaches. Traffic signals are all multiphase operating as coordinated under traffic responsive control as part of the Los Angeles central traffic control system. Loop detectors are located on each lane approximately 90 meters upstream of the intersection stop line. Detectors are also placed on the major cross street approaches.

A field study was undertaken to obtain a comprehensive database of operating conditions in the study area. Data (vehicle count and occupancy) from each loop detector were collected and stored every 30 seconds. Manual turning movement counts at each intersection were undertaken for a four hour period (6-10 am) on Wednesday May 26, 2002. The study period enabled us to attain data for a wide range of traffic conditions: from low volume off-peak conditions, peak period conditions and post-peak mid-day flow conditions. Traffic demand is high especially during the peak hour and heavily directional with the higher through and turning volumes in the northbound direction. System cycle lengths range from 100 seconds early in the analysis period (6:00 to 6:30 am) to a maximum of 150 sec during the periods of highest traffic volume (7:30 to 8:30 am).

The vehicle data and signal timing data were incorporated into the AIMSUN microscopic simulator. The proposed model was then applied to estimate the TTD on northbound travel direction. The simulation lasts 4 hours with 15 minutes pre warming. The data sampling rate from probe vehicles is every one second and the travel time of each vehicle at each link is recorded and treated as GPS data. The simulation output was first compared with field data (delays and travel times) to verify that the model reasonably replicates field conditions at the test sites. Figure 3 shows the snapshot of study site in AIMSUN.

Figure 3 Linclon Ave. snapshot in AIMSUN



5. Results and Discussion

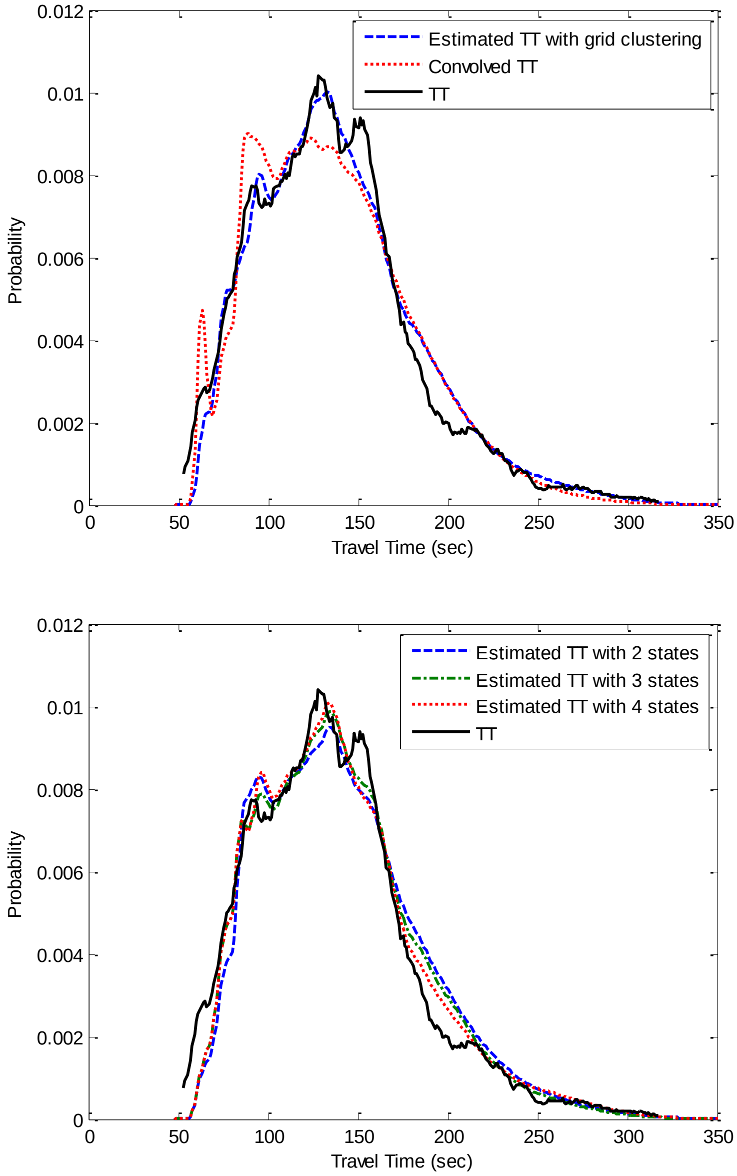
Here, results for analysis are limited to link 1 through to link 5, since Links 6 and 7 turned out to be too long (with respect to other links) and their travel times cancel out travel time of the other segment of the route. Real TTD, convolved estimation of TTD and result of our methodology with grid clustering approach are depicted in figure 4-a. The outputs of our proposed method using traffic guidelines with 2, 3 and 4 states in each link are also illustrated in figure 4-b.

Figure 4 (a) Estimation of TTD of link 1 to 5 with convolution and grid clustring method

(b) Estimation of TTD of link 1 to 5 with 2, 3,and 4 states in each link

(a)

(b)



It is apparent that the real TTD is very close to the outcomes of proposed methodology and more accurate once compared to the convolution estimation. Note that real TTD is too spiky; therefore for sake of presentation we smooth it with moving average method. Another issue about TTD being spiky is that convolution operator has the disadvantage that it generally tends to produce smooth output function contrary to our method which can produce sharp results. For more comprehensive comparison, the Mean Absolute Errors (MAE) of various methods according to (12) is calculated and summarized in table 1.

$$MAE(x, y) = \frac{\sum_{i=1}^{N} |x_i - y_i|}{N} \tag{12}$$

where N is union of estimated TTD and real TTD durations, and x_i and y_i correspond to value of estimated and real TTD, respectively.

Furthermore, different deployment rate of probe vehicles are studied too and the results are also given in table 1. The results show that the more states in markov chain procedure (more states in links), the better result is achieved. Additionally, the higher deployment rate of probe vehicles yields to more accurate estimation of TTD. Note that grid clustering algorithm deterministically detects the states depending on the spatial structure of 2D diagram. So, for different sample sizes, different number of states is selected for every link (usually around 3 states for each link). Nevertheless, our proposed algorithm still performs very well under condition of sparse probe vehicles and low number of states.

Table 1 MAE ($*10^{-4}$) of methods regarding real TTD

Probe Vehicles Sample Size	Convolution	2 States Markov Chain	3 States Markov Chain	4 States Markov Chain	Grid Clustering
100%	4.2739	4.5016	3.9789	3.8694	3.7855
50%	4.3094	4.5793	3.9990	3.9023	3.7747
20%	4.5403	4.9194	4.2587	4.1007	4.5291
10%	4.5923	4.9272	4.3770	4.3498	4.6249
5%	4.9845	5.3409	4.8056	4.6476	4.4614
2%	5.3610	5.7819	5.1530	5.3242	5.1330

The effect of probe vehicles penetration rate on evolution of TTD profile is shown in figure 5 and 6. In figure 5, TTD is estimated using 4 states in every link with a range of probe vehicles deployment. The results when merely 2% of cars are considered as probe vehicle are depicted in figure 6. Considering all the facts, incorporation of markov chain into TTD estimation in a synergy produces promising results which can capture the fundamental characteristic of field TTD entirely.

Figure 5 Estimation of TTD of link 1 to 5 with different probe vehicles penetration rate

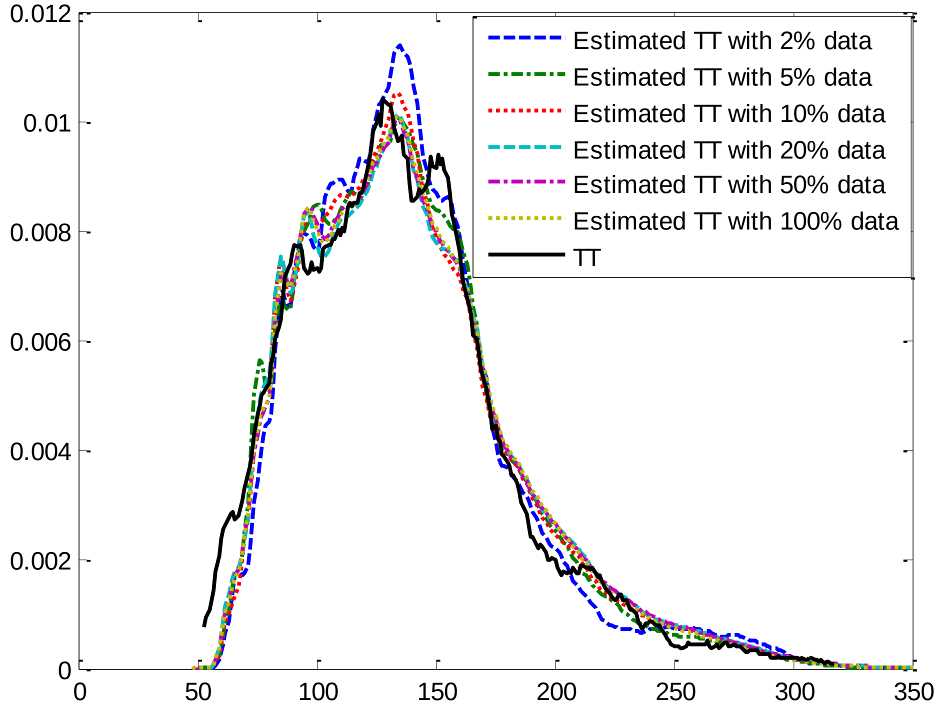
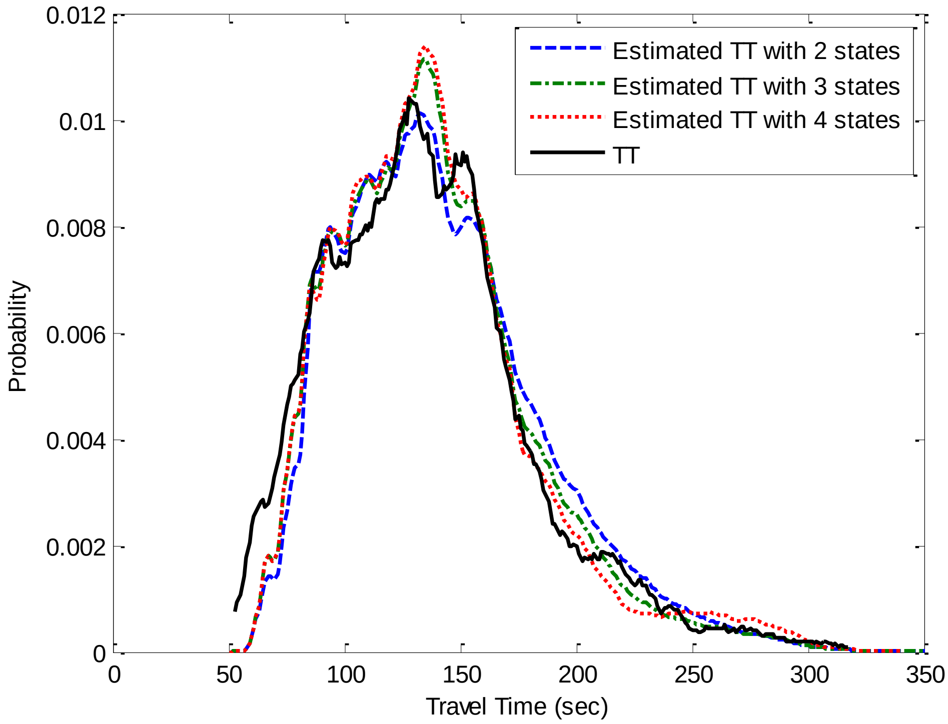


Figure 6 Estimation of TTD of link 1 to 5 with 2% probe vehicles penetration rate



6. Conclusion and Future Works

In this article, we introduce a sound approach to address traffic progression and correlation in arterials for estimation of travel time distribution. In this method, probe vehicles provide travel time of links of the arterial route. For each pairs of consecutive links, a 2D diagram is established to graphically represent the joint distributions of successive link travel times. Then using these 2D diagrams, we incorporate a Markov chain procedure into the technique and calibrate its structure. For markov chain state identification, a heuristic grid clustering algorithm is also developed. The procedure is tested with various deployment rates of probe vehicles to tackle the problem of probe vehicle sample size. Our proposed methodology shows a well performance which captures the fundamental characteristic of field measurements even under condition of sparse probe vehicles.

For future studies, the crucial step is to investigate under what conditions our method outperforms convolution estimation. This needs more empirical experiments with real data to examine influence of different traffic conditions. Moreover, this approach would be helpful to inspect how the signal timing, offset timing, site topology and etc. affect the travel time variability.

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