# Maximisation of subjective attractiveness of public transport in urban areas

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# Title of paper

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## Abstract

The quality of a public transport system depends mainly on the configuration of its network including service density. Planning attractive and efficient networks for public transport in densely populated areas therefore requires knowledge of fundamental principles of network design.

This paper presents the derivation of network design principles for public transport in urban areas on the basis of a heuristic model. With this model the maximum of subjective attractiveness of public transport networks is searched under the constraint of a fixed number of vehicles in service.

Attractiveness is measured by three supply quality criteria: spatial, temporal availability, and door-to-door travel time. Subjective attractiveness of a public transport system is the reaction of transport demand to changes in the three supply quality criteria. This involves estimating transport demand with a probabilistic model.

The model aims an optimal choice of the network design variables route and link density and headway. Therefore measurable terms of the network design variables, such as link spacing and headway are considered.

The paper concludes that a network for public transport should consist of a few routes with high speeds and short headways. Public transport supply is efficient if transport demand in a developed area exceeds a minimum value. An efficient public transport system therefore requires a spatial pattern of land use that is adapted to its network structure.

## Keywords

Public transport, Supply modelling, Network design, Heuristic models

# 1. Background

## 1.1 Motivation

The quality of public transport systems is mainly determined by its network. Due to that, network design is one of the main businesses in planning of public transport systems.

This paper aims to formulate fundamental principles for the network design of public transport networks. That is the derivation of basic interrelation between the network design variables with general validity. Its intention is providing the planer of public transport networks guidelines for the design of efficient networks with a maximal level in quality.

## 1.2 Network design problem

The network design problem is related to the questions:

How does an optimal network look like and

How can "optimal be defined in this context?

In the approach here, a network design is optimal, when its acceptance by its users attains a maximum. Therefore, network designs must be comparable and consequently based on an equal level in operation expenses. A simple manner to meet this requirement exists in the assumption of a fixed number of vehicles in service per unit of area in each of the network designs.

The acceptance by its users is specified in this approach in two different ways:

1. Maximisation of the factual supply quality of a public transport system
2. Maximisation of the subjective quality of a public transport system

The factual supply quality is measured by the unweighted door-to-door travel times in a certain network design. This approach does not consider subjective experiences of travellers, which exist for example in the perception of different travel time components, like waking time or access time (Hoogendoorn-Lanser und Hoogendoorn (2002).

In opposite to this, in the second approach human perceptions are taken into consideration and acceptance of public transport networks are measured by the share of ridership in public transport.

### 1.3 Objective

In paper aims to answer the quest, whether the principles for optimal network designs, which follow form these two approaches described above, are identical or different in their conclusion.

## 2. Model

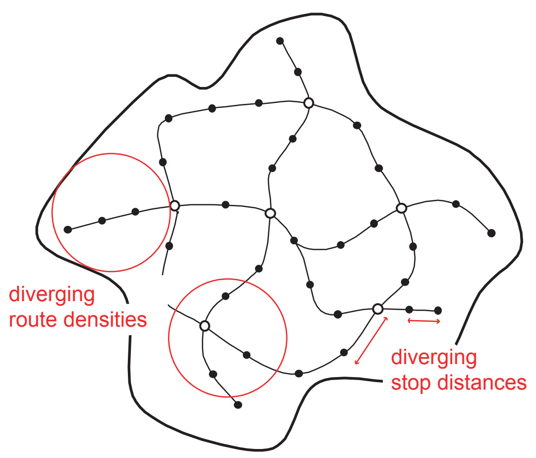
### 2.1 Assumptions

The optimisation is based on heuristic models used by van Nes (2002) and Jung (1996), where public transport networks are described by mathematical formulas. Networks are characterised by specific network design variables that are:

- Stop spacing,
- route spacing,
- line density and
- headway.

While the values of these network design variables are not consistent in real public transport networks like it is shown in Figure 1, an idealised network model is necessary that fits to this requirement.

Figure 1 The values of the network design variables vary within a network.

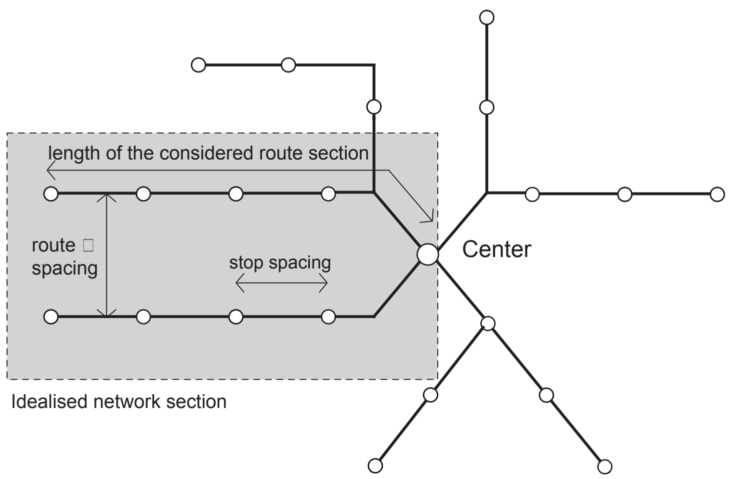



Moreover, the idealised network model has to be based on certain assumptions that are :

- the public transport route are parallel
- settlement density in the considered area is constant
- the amount of transport demand depends on the level of its quality
- supply density is constant in the considered area, that significates
    - stop spacing is constant,
    - headway is constant,
    - on each route only one public transport line is in operation.

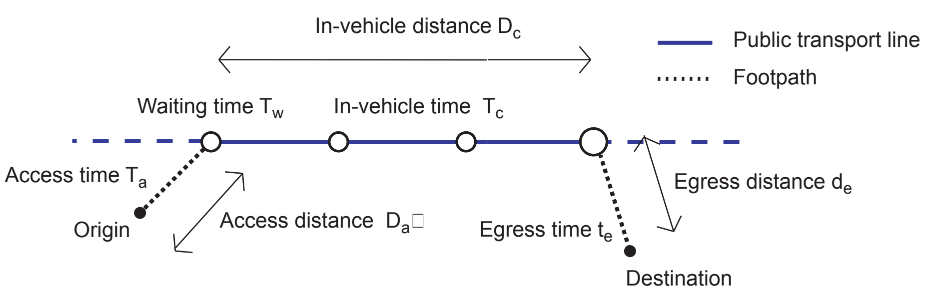
Figure 2 illustrates the assumptions of the model.

Figure 2 Idealised network section




The network design variables have constant values within the idealised network section. This allows determining supply quality analytically with in the section.

Figure 3 Model for the calculation of door-to-door travel time



### 2.1.1 Access Time

The linear modelling of access distance depends on the chosen network structure (see chapter 3). However, access distance must be proportional to the sum of the two spatial network design parameters, stop distance and route path distance.

$$D_a = \delta_a \cdot \left(D_s + D_l\right) \tag{1}$$

For parallel routes paths, the network geometry coefficient $\delta_a$ can be specified with one quart (two times the half of stop and route path distance).

$$\delta_a = 0.25 \tag{2}$$

As a result, access time is a function of access distance and agrees velocity:

$$T_a = \frac{\delta_a \cdot \left(D_s + D_l\right)}{v_a} \tag{3}$$

### 2.1.2 Waiting time

In urban networks, the waiting time $T_w$ for a service can be approximated as half of the service alternating time. The alternating time itself it the reciprocal value of frequency.

$$T_w = \frac{1}{2 \cdot F} \tag{4}$$

### 2.1.3 In-vehicle time

The transportation time between two stops is determined by the acceleration time of a vehicle, the transportation time with the transportation velocity $v$ and the deceleration time. Furthermore, the devel time at a stop $T_{ba}$ has to be added.

$$T_s = \frac{D_s}{v} + \frac{v}{a} + T_{ba} \tag{5}$$

### 2.1.4 Egress Time

Egress time $t_e$ is assumed to be constant in the approach here.

### 2.1.5 Door-to-door travel time

The non-weighted travel time is consequently the sum of the durations of above listed parts of a trip:

$$T_t = T_a + T_w + T_s + t_e \tag{6}$$

or:

$$T_t = \frac{\delta_a \cdot (D_s + D_l)}{v_a} + \frac{1}{2 \cdot F} + \frac{D_c}{D_s} \cdot \left( \frac{D_s}{v} + \frac{v}{a} + t_s \right) + t_e \tag{7}$$

### 2.1.6 Operations costs

The number of vehicles in service expresses operation costs $K_o$. The number of vehicles $Z_o$ within a determined area has to be fix, $k_o$ expresses the operation costs of one vehicle within a certain time period.

$$K_o = k_o \cdot Z_o \tag{8}$$

The vehicles are spread on the route paths in this area (route path distance $D_l$). On a route path, if there is only one line going along on each route path), the number of vehicles is determined by the vehicle following distance $D_o$.

$$Z_o = \frac{1000\ m}{D_l} \cdot \frac{1000\ m}{D_o} = fix \tag{9}$$

The vehicle following distance is calculated as follows:

$$D_o = \frac{D_s}{F \cdot T_s} = \frac{D_s}{F \cdot \left( \frac{D_s}{v} + \frac{v}{a} + t_s \right)} \tag{10}$$

The cost budget can be formulated:

$$K_o = k_o \cdot \frac{(1000\ m)^2}{D_l \cdot D_s} \cdot F \cdot \left( \frac{D_s}{v} + \frac{v}{a} + t_s \right) \tag{11}$$

## 3. Maximisation of factual supply quality

Door-to-door travel time is a suitable indicator for the quality of a transportation network, as it is a function of all network design variables like one can see in Equation (7). Consequently, the minimisation of door-to-door travel time leads to an optimisation in consideration of all network design variables. Travel time has with regard to frequency no minimal value, because travel time is a linear function of it. Therefore, the optimisation is limited to the variables stop spacing and line distance.

From formula (7) and (9) follows for the optimal values for the network design variables:

$$T_t = \frac{\delta_a \cdot (D_s + D_l)}{v_a} + \left( \frac{(1000\ m)^2}{2 \cdot Z \cdot D_l \cdot D_s} + \frac{D_c}{D_s} \right) \cdot \left( \frac{D_s}{v} + \frac{v}{a} + t_s \right) + t_e \qquad (12)$$

The optimal stop spacing leads to a minimal value for travel time in function (12). This minimum follows from:

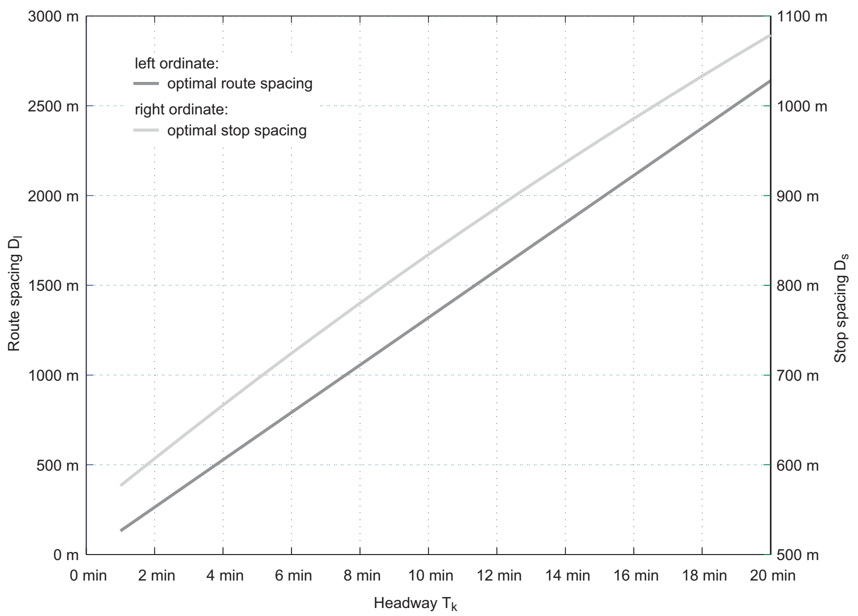
$$\frac{dT_t}{dD_s} = 0 \qquad (13)$$

The resulting term for stop spacing cannot be analytically written in a simple form. For route spacing, the optimal term ends in a simple expression with headway as the only remaining network design variable.

$$D_l = 2 \cdot \delta_a \cdot \frac{T_k}{v_a} \qquad (14)$$

With regard to function (14), optimal values for route spacing are a linear function of headway. Figure 4 illustrates this linear dependency between headway and optimal route spacing. Furthermore, optimal values for stop spacing against headway have an approximate linear shape.

Figure 4 Optimal route and Stop spacing




The values obtained in this optimisation for stop spacing are pretty high compared to the values found in practise. Average stop spacing for tram- und bus lines in the city of Zurich are about 350 m. A case study for the tramway network of Den Haag has shown, that stop spacing beyond 600 m cannot be reasonably adjusted with the existing land use (Schäffeler, 1999). Furthermore, willingness to ride by public transport increases disproportionate with the distance to a stop (Walter, 1973).

Therefore, travellers behaviour will be taken into consideration in a second approach for the optimisation of the network design variables stop and route spacing.

# 4. Maximisation of subjective supply quality

## 4.1 Approach

### 4.1.1 Choice Model

The reaction of travel demand on a certain transport system is measured in the approach here with a choice model for different transport alternatives. Therefore, two alternatives are taken into account, a public transport system ($\ddot{o}V$), based on the investigated network design variables and private vehicle traffic ($IV$) as competitive transport system. The optimisation subsequently aims to maximise the share of public transport usage:

$$P_{\ddot{o}V} = \frac{exp(N_{\ddot{o}V})}{exp(N_{\ddot{o}V}) + exp(N_{IV})} = max \tag{15}$$

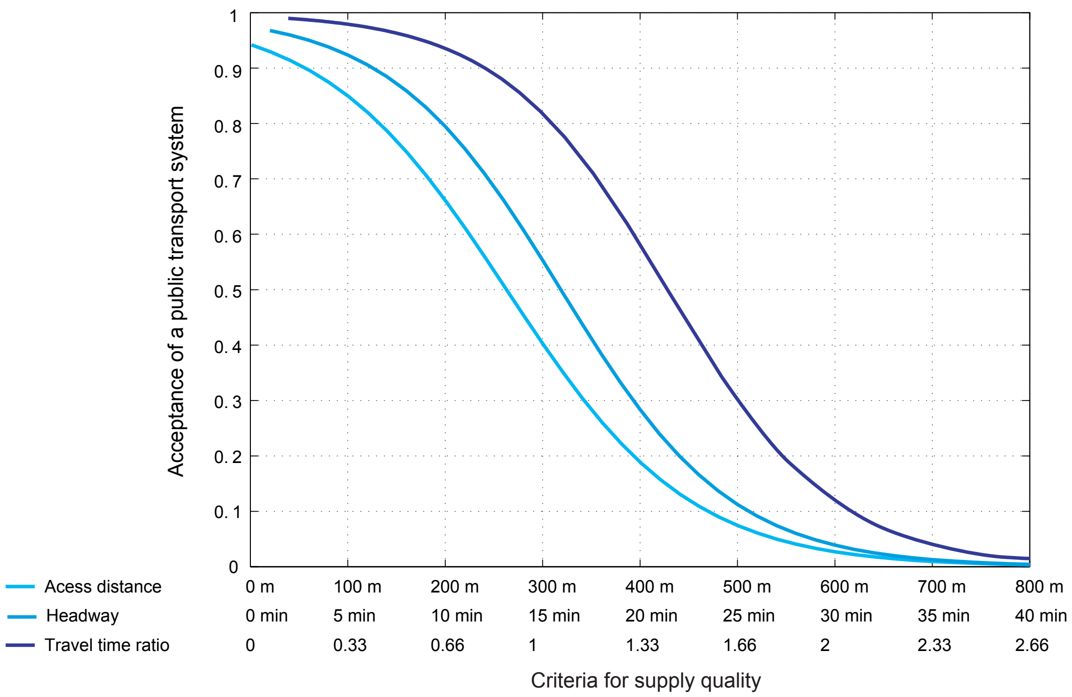
### 4.1.2 Synthetic Utility Function

Only the network design related quality criteria are considered in the utility function $N$. All other criteria, especially private vehicle transport related criteria, are subsumed in a constant value $C_j$.

$$N_j = C_j + (\alpha_{D_a} \cdot T_{a_j}) + (\alpha_{T_k} \cdot T_{w_j}) + (\alpha_{T_t} \cdot T_{t_j}) \tag{16}$$

The estimation of the parameters of the utility function is not based on data from a specific transport survey but on general assumptions about travellers' behaviour. This approach allows testing the results of the optimisation for their sensitivity towards variation in travellers' behaviour.

Figure 5: Assumptions for travellers' behaviour with regard to the quality criteria of a public transport network



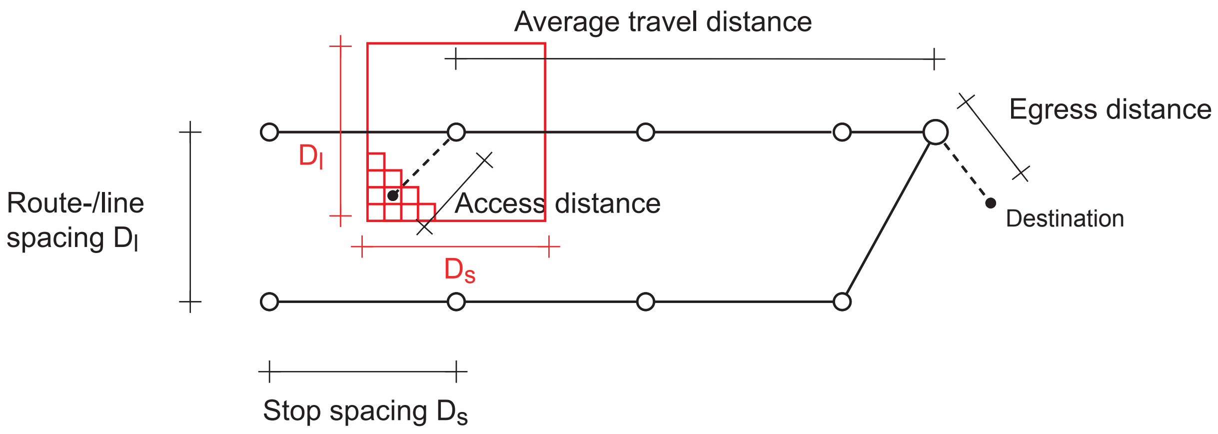

## 4.2 Optimisation

For the optimisation, the average share of public transportation usage of an entire development area is searched. The share of public transport usage is an exponential function of the supply quality criteria. Thus, average values for e.g. access distances do not satisfy for a calculation of public transport usage in a development area of a stop.

Consequently, an integration over the area, like it is expressed in function (17) and illustrated in Figure 6, is necessary.

$$P_{\varnothing}(G) \approx \frac{1}{D_l \cdot D_s} \cdot \int_{D_l} \int_{D_s} P_{\ddot{o}V}(x,y)\,dxdy \qquad (17)$$

Figure 6 Integration over a development area of a stop.



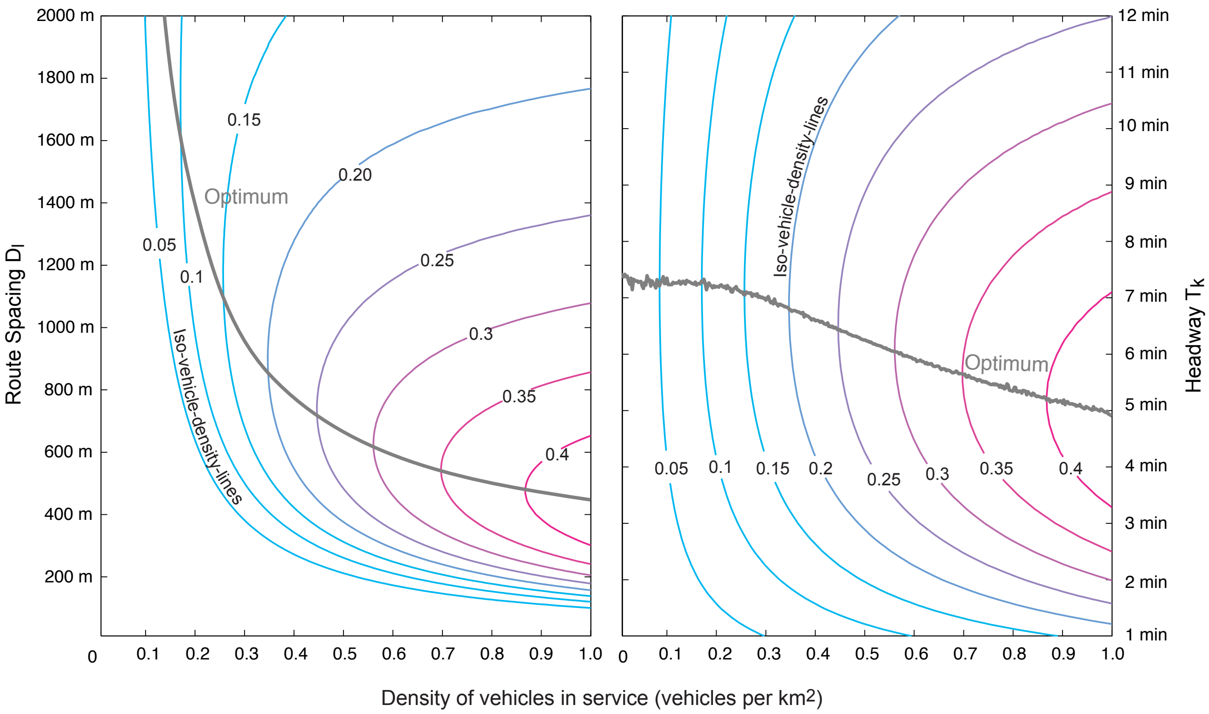

To get optimal values for the network design variables stop spacing, route spacing and headway the maximum in share of public transport usage in dependency of these network design variables is calculated by searching the null of the corresponding function (function (18)).

$$\frac{\partial P_{\varnothing}(G)}{\partial D_s} = 0 \quad und \quad \frac{\partial P_{\varnothing}(G)}{\partial D_l} = 0 \qquad (18)$$

## 4.3 Results

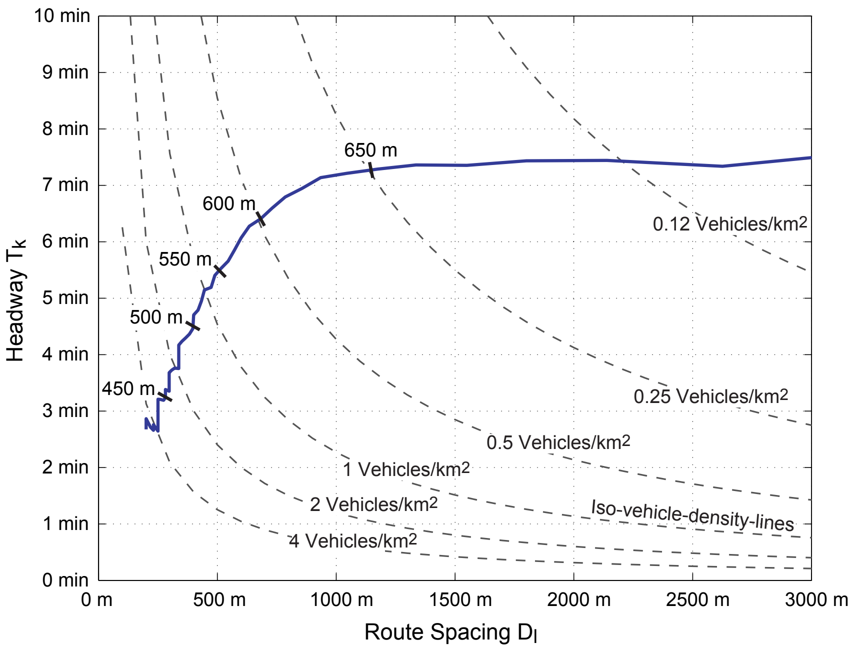
Figure 8 shows the optimal values for route spacing and headway against variation in the number of vehicles in operation. Regarding this, the optimal value for route spacing crucially depends on the number of vehicles in operation, which is the specific supply density of a network. In contrary to that, the optimal value for headway is almost independent from the number of vehicles in operation.

Figure 7 Shape of optimal values for route spacing (left) and headway (right) against the density of vehicles in operation




The optimal value for headway does not exceed a certain maximal value (in the example here ~7.5 min). This results also from Figure 8, where optimal headway is plotted against optimal route spacing for varying vehicle densities. Furthermore, Figure 8 shows the optimal values for stop spacing that varies only in a small range from 500 m to 650 m.

Figure 8: Optimal headway against optimal route spacing for increasing vehicle densities. The numbers written along the curve are the optimal values for stop spacing.




# 5. Conclusion

## 5.1 Comparison of the results

The results of the optimisations provide quiet different results, depending on the chosen approach. Values for stop spacing delivered by the maximisation of factual supply quality are lower and therefore closer to values realised in today's public transport networks.

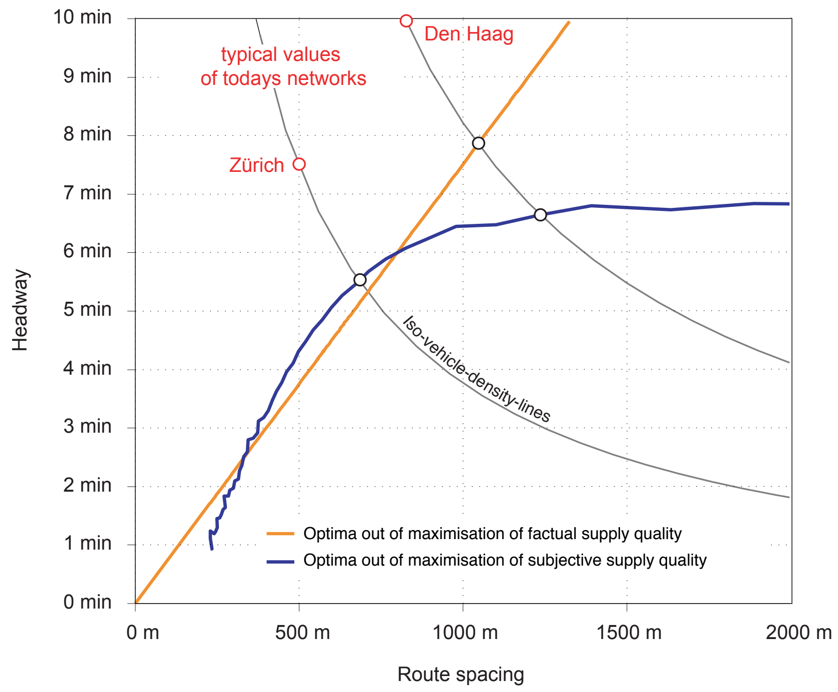
Regarding route spacing and headway, the maximisation of subjective supply quality delivers fundamental different results than the minimisation of the door-to-door travel time. In the first approach, headway against route spacing is linear, whereas in the second approach this relation has an asymptotic curve. Accordingly, headways of public transport systems shouldn't exceed a certain maximal value (in the illustration here 7.5 min).

Taking only objective supply quality into account leads to a network quality that does not fit to how users experience it.

## 5.2 Consequences for the design of public transport networks

The values for the network design variables fond in this investigation lead to public transport system with less spatial density than it is common in network design today. Optimal values for stop spacing are in a range of 500 m and more. Route spacing should be increased in favour of high frequencies. Headway optimally should not be higher than 10 minutes. Figure 9 show the values of network variables of today's networks and in comparison the corresponding optimal values found in this investigation.

Figure 9: Values for route spacing and headway found in today's public transport networks, and in comparison, the corresponding optimal values.




Below a specific supply density attractiveness of public transport is higher when its supply is concentrated on corridors with high demand, where frequencies remain on the required high level. In exchange, the development of corridors with lower demand should be abandoned.

Consequently planning of public transport networks should focus much more on a high temporal density and less on its spatial density.

# 6. Literature

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